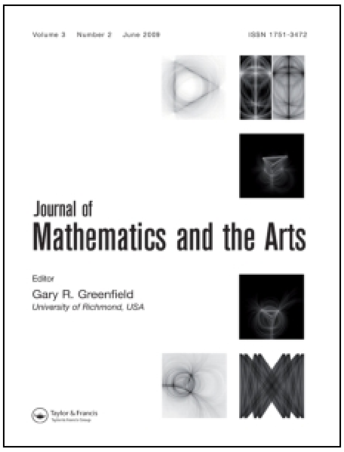

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Modelling with poetry in an introductory college algebra course and beyond

Sarah Glaz [a]; Su Liang [a]
[a] Department of Mathematics, University of Connecticut, Storrs, Connecticut, USA

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Modelling with poetry in an introductory college algebra course and beyond

Sarah Glaz* and Su Liang

Department of Mathematics, University of Connecticut, Storrs, Connecticut, USA

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The main focus of this article is the pedagogical use of poetry in a college course, Introductory College Algebra and Mathematical Modelling, which was designed to prepare students with weak mathematics background for science courses. We use poetry projects to ease the difficulties students have with the transition between word-problems representing natural phenomena, and the corresponding mathematical models – the equations representing the phenomena. We also use poetry projects to stimulate classroom participation, develop mathematical intuition, enhance number sense and introduce new concepts in an engaging setting. We present two examples of group-projects that we employed in our course, as well as a number of other poems which may be used to construct additional group or individual projects in courses of similar mathematical level. We conclude by providing a brief survey of the pedagogical uses of poetry in the K-14 mathematics classroom, and examples of poetry that may be used to enhance learning experience in calculus classes.

Keywords: poetry; intermediate algebra; mathematical modelling; Bhaskaracharya; Martin Gardner; calculus

AMS Subject Classifications: 97D30; 97D40; 97D20

1. Introduction

In the academic year 2006–2007 the first author of this article developed a new course at the University of Connecticut, Math 1011Q: Introductory College Algebra and Mathematical Modelling [10]. This course was designed in response to requests from science departments to provide a mathematics course that will offer an engaging and effective preparation for science courses for students whose high-school algebra needs reinforcement. Taking these needs into account, the course strikes a balance between covering the necessary material and practicing its applications, and thus provides both the body of knowledge, and the skills, necessary for solving multi-step problems from other disciplines. We use a college algebra approach to cover the material of an expanded intermediate algebra course. The practice of applications is accomplished through a substantial component of mathematical modelling. Students engage in one or two group-projects in mathematical modelling or related topics, every week throughout the semester. Constructing group-projects and experimenting with implementing them in the classroom is an ongoing activity carried out by the team of instructors who coordinate or teach sections of this course. The second author joined the team soon after the course became permanent, and together we experimented with a variety of group-projects.

The first poetry group-project we introduced was *Lilavati's Swarm* (Section 2). Our original intention was to reach out to the students in this class, most of whom intend to major in the humanities, through a medium they know and love – the language of poetry. We timed this group-project to coincide with the first topic students have difficulties with, the topic of word-problems. And indeed, we were gratified to see that the project seemed to inspire students, and stimulate enthusiastic participation. We soon discovered that the language of poetry serves other important purposes as well. It eases the difficulties students have with the transition between word-problems representing natural phenomena, and the corresponding mathematical models – the equations representing the phenomena in 'mathematical language'. The language of poetry also helps develop mathematical intuition, and number sense. The second poetry group-project, *How Old is the Rose-Red City*? (Section 3), was introduced to reinforce the lessons learned from the first project, but like its predecessor it turned out to have additional uses. This group-project exposes students to a more difficult word-problem, whose solution involves a higher level of pattern recognition – an important mode of mathematical thought, as well as a technique for problem solving in mathematics. It also allows for the introduction, in a natural and engaging setting, of a mathematical

*Corresponding author. Email: glaz@math.uconn.edu

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concept that appears frequently in mathematical modelling, the concept of a 'unit'. We elaborate on these two projects and their pedagogical uses in Sections 2 and 3 of this article. These sections also contain handouts of both projects for instant classroom use, and provide a number of sources for additional poetry projects in courses of similar mathematical level. Section 4 provides a brief survey of the pedagogical uses of poetry in the K-14 mathematics classroom, placing our work in the context of other efforts made by educators in this direction. This section also contains examples of poetry which may be used to enhance students' learning experience in calculus classes.

The poetry projects we used in the course, Math 1011Q: Introductory College Algebra and Mathematical Modelling, contributed to improved students' attendance and performance, and a decrease in math-anxiety. In the concluding section, we present a statistical analysis indicating initial signs of success.

2. Mathematical poetry from *Lilavati*

The first poetry group-project we introduced in this course, *Lilavati's Swarm* (Table 1), contains the translation from Sanskrit of a mathematical poem by one of the best known mathematicians of ancient India, Bhaskara (1114–1185). Its source adds to its appeal by putting mathematics in historical and social context. For a biography of Bhaskara, also called Bhaskaracharya, and his mathematical legacy, see [12], [13], or [34]. The historical quotation in this group-project comes from [13]. The translation of the poem is taken from [32].

We introduced this project just after finishing the word-problems section to replace a number of more conventional homework problems. The project reinforces the four traditional steps students are advised to follow to understand and solve a word-problem. The naming of the steps: UNDERSTAND, TRANSLATE, SOLVE and INTERPRET, due to Martin-Gay [23], emphasizes the fact that mathematical modelling is just 'a translation' of real-life phenomena into mathematical language, and after manipulating the mathematical language and obtaining a mathematical solution, one needs to interpret this solution into the real-life language of the original problem.

We added a 'reflective' TRIAL AND ERROR component under the UNDERSTAND step. This component is intended to develop students' ability to think mathematically, their number sense and mathematical intuition. It also embodies the trial and error aspect of the mathematical modelling process itself. The TRIAL AND ERROR phase encourages students to try various numbers as solutions to the problem posed in the poem, and to reflect on their findings. Reflection, and the ensuing group discussion, alerts students to emerging patterns. Patterns appear in both poetry and mathematics. In poetry, it may be a factor distinguishing a poem from a prose piece. In mathematics, recognizing a pattern is often an important step towards a solution of the mathematical problem. Moreover, mathematical ideas develop through the interplay between the concrete (sometimes numerical) examples, and the abstraction which is the generalization of these examples. Therefore pattern recognition, which often assists in the passage from the concrete to the abstract, is an integral component of the mathematical way of thinking. In addition to assisting students to set up the equation and solve the particular problem, reflective trial and error also trains them to think mathematically.

In the *Lilavati's Swarm* project, the TRIAL AND ERROR phase encourages students to try various numbers for the total number of bees in the swarm, and reflect on emerging patterns. They realize pretty quickly that no fraction of a bee can fly and so the number of bees in the swarm has to be divisible by both five and three. From here a small step brings them to the realization that the number of bees in the swarm must be 15. The TRIAL AND ERROR phase, when accompanied by reflection and alertness to emerging patterns, bridges the gap between the concreteness of the numerical solution, and the abstractness of the equation they need to set up in the TRANSLATE phase. It makes students discover on their own that the variable x in the equation must be the unknown they have looked for numerically in the TRIAL AND ERROR phase, that is, in this case, the number of bees in the swarm; and that the equation they set up needs to follow the steps for x, that they followed for each of the individual numerical values they tried.

Bhaskara's book, *Lilavati*, was written entirely in the form of poems. An 1817 translation by H.T. Colebrooke is available as a 1993 reprint by the Asian Education Services [6]. Below are two poems from *Lilavati* found in [6]. The translation of both poems is taken from [17].

from Lilavati: **How Many Lotus Flowers?**

by Bhaskaracharya

Of a cluster of lotus flowers,
A third were offered to Shiva,
One fifth to Vishnu,
One sixth to Sûrya.
A quarter were presented to Bhâvanî.
The six remaining flowers
Were given to the venerable tutor.
How many flowers were there altogether?

Table 1. Poetry project: Lilavati's Swarm.

Group project on linear equations: How Many Bees in Lilavati's Swarm?

In twelfth century AD there lived in India a famous mathematician by the name Bhaskara, or Bhaskaracharya. His contributions to mathematics were so remarkable that a medieval inscription in an Indian temple reads: 'Triumphant is the illustrious Bhaskara whose feats are revered by both the wise and the learned. A poet endowed with fame and religious merit, he is like the crest on a peacock.'

And indeed, as it was the custom in India of his days, all of Bhaskara's mathematical works were written in verse. His most charming book, *Lilavati*, written for his daughter, whose nickname was Lilavati (meaning 'the beautiful'), contains many interesting algebraic poems. Apparently beautiful Lilavati, following in her father's footsteps, had a taste for higher mathematics. Here is one poem from *Lilavati**:

A fifth part of a swarm of bees came to rest
on the flower of Kadamba,
a third on the flower of Silinda.
Three times the difference between these two numbers
flew over a flower of Krutaja,
and one bee alone remained in the air,
attracted by the perfume of a jasmine and a bloom.
Tell me, beautiful girl, how many bees were in the swarm.

Follow the four steps followed by Lilavati to find the answer to the question posed in the poem:

1. UNDERSTAND the problem thoroughly.

Read: read and reread the poem.

Trial and Error: Check if a few arbitrary values give you a solution. For example, check if a swarm of 6 bees satisfies all the conditions of the poem. Pick your own additional values for the number of bees in the swarm, and try them out. *Reflect on your answers*. After three or more trials make a guess of what the solution will be.

2. TRANSLATE the problem into an equation.

Choose a variable to represent the unknown: Let $x =$

Write an equation in x for your problem:

3. SOLVE the equation for x.

4. INTERPRET the solution.

Check your solution:

State your answer in words:

*Poem excerpt is reprinted from *The Man Who Counted*, by Malba Tahan, translated by Leslie Clark and Alastair Reid. © 1972 by Helio Marcial de Faria Pereira. Translation © 1993 by Leslie Clark and Alastair Reid. With permission of the publisher, W.W. Norton & Company, Inc.

The second poem is a later addition, of uncertain period and by a different author, to the original manuscript of *Lilavati* [6]. It also appears in [12], and online at [36]. It may make an entertaining poetry project for a class of adult returning students:

> *from Lilavati*: **A Mathematical Problem**
>
> *by Rama Krishna Deva*
>
> Whilst making love a necklace broke.
> A row of pearls mislaid.
> One sixth fell to the floor,
> One fifth upon the bed;
> The young woman saved one third of them;
> One tenth were caught by her lover.
> If six pearls remained upon the string
> How many pearls were there altogether?

The poems from *Lilavati* are particularly suited to maximize the benefits students may draw from poetry projects in a course of this level. The poems possess the elements necessary to imprint the lessons learned in the process of solving them to memory. These elements are number, rhyme and image, or – generalizing from the concrete to the abstract – mathematics, poetry and visualization. In order to understand better the ‘imprinting to memory’ effect of combining these elements we will take a short detour through the literature.

The phenomena of memorization of mathematical entities involving rhyme or poetic meter is of long standing. We are familiar with the many amusing mnemonics used to memorize π, e and other special numbers. See, for example, [31] for an entertaining collection of mnemonics based on rhyming schemes. We also used such a mnemonic in our course to help students memorize the quadratic formula. The rhyme below, by an unknown author, appeared in the 1885 Proceedings of the Edinburgh Mathematical Society [38] (it also appears in [20], and [31]):

> From square of b take $4ac$;
> Square root extract, and b subtract;
> Divide by $2a$; you’ve x, hooray!

Rhymes and numbers appearing in the same mnemonic devise assist in memorizing a list of objects. One of the earliest mnemonic devices appearing in modern psychological literature, used for memorizing a list of objects, employs the nursery rhyme below [25].

> One is a bun,
> Two is a shoe,
> Three is a tree,
> Four is a door,
> Five is a hive,
> Six is sticks,
> Seven is heaven,
> Eight is a gate,
> Nine is a line, and
> Ten is a hen.

This rhyme may be used to memorize a list of ten objects in the following way: first memorize the poem. Second, order the ten objects in the list from one to ten. Next, use visualization to link objects to the poem’s lines as follows: for the first object in the list visualize an image connecting it to a ‘bun’, for the second object in the list visualize an image connecting it to a ‘shoe’ and so on till the list is exhausted. The association of objects to images in numerical order makes memorization almost automatic. Levine [21, chapter 5] gives a more detailed account of this memorization method, and a review of the psychological literature connecting visualization to memorization. In particular, he points out a study [5], carried out with groups of college students, affirming that ‘picturing means remembering’, and consequently, visualizing enhances both learning and retention. Moreover, the more vivid, outrageous, or unusual the image the more lasting the memory, a fact that was reconfirmed in studies involving memorization of words in a foreign language [4,5,21].

The poems in *Lilavati* possess poetic meter, mathematics and numbers and a vivid, exotic and unusual imagery – all the elements that facilitate memorization of the lessons learned while solving the problems posed in its poems. We noticed that students were more aware than we would expect in a course of this level, of how their solution fits with the content of the problem they solved. A few weeks after the *Lilavati’s Swarm* project, students were engaged in a life expectancy group-project in which they had to interpret the meaning of the slope and y-intercept of a life expectancy line, and we were surprised by the number of students in class who grasped the meaning correctly. Later in the semester, students were engaged in a group-project where they had to solve a quadratic equation to obtain 'time'. Once again, we noted that no one needed a reminder to discard the negative solution. Neither have we seen the ubiquitous negative area as an answer to any area problem we assigned in this course.

3. The Rose-Red City project and other poetry sources

The second poetry group-project we introduced in this course, *How Old is the Rose-Red City?* (Table 2), is an adaptation of a puzzle-poem by Martin Gardner [9]. The poem, without the preamble, also appears online at [35]. It features one memorable line from a poem by the English poet John W. Burgon (1813–1888), and an appealing humorous presentation. Students work on this project immediately following the first poetry project.

This group-project reinforces the lessons learned in the first project at a higher level of difficulty. Note that we suggest starting the TRIAL AND ERROR phase at a city age of 3 billion years, which means that students need to do five trial and error cases to reach the solution of 7 billion years. The larger number of trials, larger than those needed for the *Lilavati's Swarm* project, has a purpose. While trying various numbers, and reflecting on emerging patterns, students become very familiar with the requirements of each of the poem's lines, a necessity for setting up this, more difficult, equation. The suggested table in which students plot each TRIAL AND ERROR result provides signposts for what to look for, and helps organize the numerical data. But we also had another purpose for introducing this table. The recording of the repeated trial and error cases in the table gives students sufficient time, and a visual aid, to notice that a solution needs to satisfy the following pattern:

(value in the 2^{nd} column and 3^{rd} row)
$= (2/5) \times$ (value in the 3^{rd} column and 4^{th} row).

This observation helps students discover the equation they need to set up for this problem. Moreover, through the trial and error process of the *Rose-Red City* project students practice a higher level of pattern recognition than needed for the *Lilavati's Swarm* project.

In addition, this group-project introduces the concept of 'a unit', which may not necessarily have length equal to one, in a natural and engaging way. No one questions the fact that '1 billion' is the 'best suited' unit to work with in this project. The next group-project is an environmental mathematical modelling group-project involving analysis and graphing of real data related to a hurricane, including wind speed, hurricane's path and predicted and real damages. The various unit sizes needed for graphing purposes for the 'pictures' to be sufficiently visible on the page to allow for the drawing of conclusions, come naturally after students have seen the possibility of changing unit size in this poetry group-project.

Note that this particular project may also be used as a poetry project when introducing solutions of a system of two equations with two variables.

J.A.H. Hunter (1902–1986), who wrote a syndicated newspaper column, *Fun with Figures*, in the mid-twentieth century, collected his entertaining mathematical puzzles in [15] and [16]. Several of these puzzles are light and lively poems that may be used for projects in courses of similar mathematical level. A number of his puzzle-poems require the finding of solutions for two equations with two unknowns. Here is one such poem from [16]. A few other poems of this kind appear in [12], and [15].

An Ugly Monster

By J.A.H. Hunter

A freak, a most unusual pike,
Was caught at Jackson's Point by Mike.
This fish was ugly, huge and strong,
With head alone twelve inches long.
Its body equalled, so Mike said,
Just half its tail plus twice its head:
A third the monster's total length
Was tail, grotesque, but built for strength.
So now maybe you'd like to see
How long this curious fish would be.

Hunter's books include a number of poems on a different topic related to solving equations with two unknowns – the topic of solving for one of the variables in terms of the other variable. Particularly, these poems result in one linear equation in two variables x and y, where x and y are known to be positive integers. The solution to the equation is found by first solving for one of the variables in terms of the

Table 2. Poetry project: How Old is the Rose-Red City?

Group project on linear equations: How Old is the Rose-Red City?
(Adapted from: *My Best Mathematical and Logic Puzzles*, by Martin Gardner)

Two professors, one of English and one of Mathematics, were having a drink at the faculty club. 'It is curious,' said the English professor, 'how some poets can write one immortal line and nothing else of lasting value. John W. Burgon, for example. His poems are so mediocre that no one reads them now, yet he wrote one of the most marvellous lines in English poetry: '*A rose-red city half as old as Time*''.

The mathematician, who liked to annoy his friends with improvised brainteasers, thought for a moment or two, then raised his glass and recited:

A rose-red city half as old as Time.
One billion years ago the city's age
Was just two-fifths of what Time's age will be
A billion years from now. Can you compute
How old the crimson city is today?

The English professor had long ago forgotten his algebra, so he quickly shifted the conversation to another topic. But you are eager to practice your algebra. Follow the four steps listed below to answer the question posed in this poem:

1. UNDERSTAND the problem thoroughly.

Read: Read and reread the poem.

Trial and Error: Check if a few arbitrary values give you a solution. For example, check if a rose-red city age of 3 billion years satisfies all the conditions of the poem. Pick your own additional values for the city's age, and try them out. *Reflect on your answers*. After three or more trials make a guess of what the solution will be. You may organize each trial in the following format:

	City's age (in billion years)	Time's age (in billion years)
Present: Today	3	
Past: One billion years ago		
Future: One billion years from now		

Is the city's age a solution?

2. TRANSLATE the problem into an equation.

Choose a variable to represent the unknown: Let $x =$

Write an equation in x for your problem:

3. SOLVE the equation for x.

4. INTERPRET the solution.

Check your solution:

State your answer in words:

other. Here is a small sample of poems of this kind from [15]:

The Spider and the Fly

by J.A.H. Hunter

'Come right into my parlour', said
The spider to the fly,
'and answer one small question, please.
Unless you want to die.
I've eaten scores of flies, of course,
But tell me if you dare:
If females had two more, and males
But half their present share,
How many flies like that, d'you think,
I really would require,
To give me twenty-eight fly legs,
The number I desire?'

Students may need to be told that flies have six legs. If x denotes the number of female flies, and y denotes the number of male flies, the poem requires solving $8x+3y=28$. Solving for y in terms of x, students obtain $y=(28-8x)/3$. Since both y and x are positive integers a trial and error check shows that $x=2$, and $y=4$.

A Tale of the Cats

by J.A.H. Hunter

The lucky cats in Stratton Street
Had seven mice apiece to eat.
  The rest made do
  With only two:
  The total score
  Being twenty-four.
How many cats ate mousie meat?

If x denotes the number of lucky cats, and y denotes the number of the rest, the poem requires solving $7x+2y=24$. The answer is seven cats.

Several other of Hunter's algebraic poems are available online at [29].

4. Poetry in the K-14 mathematics classroom: a brief survey and a few suggestions for calculus classes

The time of *Lilavati*'s India, when mathematical knowledge appeared in verse form, is long past. Nevertheless, poetry retained its uses as an educational tool for teaching mathematics at the K-14 levels through the ages. The nature and frequency of this use fluctuates to reflect technological advances, and changing attitudes to mathematics education. To place our work in the context of other efforts made by educators to use poetry for teaching mathematics, we offer below a brief survey of the literature, and point out selected resources. We conclude this section with examples of poetry that may be used to enhance students' learning experience in calculus classes.

Everyone's mathematical education starts with counting rhymes such as, for example, *One is a Bun*, which appears in Section 2. The abundance of counting rhymes found on the internet is a testimony to their perennial appeal and effectiveness as memory devices for young children. *Scholastic*: *Max's Math Adventures* [30] is an interesting site which offers, in their own words, 'math and language games', focusing on mathematical skills integral to the K-2 math curriculum. In addition to counting rhymes (K level), there are mathematical problem-poems, whose solutions, carried out under teacher guidance, require students to use creative thinking and grade appropriate mathematical skills. Another interesting internet site, *Tooter4kids* [37], provides mathematical problem-poems appropriate for grades 3–8. *Tooter4kids* contains the delightful poem, *Smart*, by Shel Silverstein (also found in [12]). Other noteworthy sources of mathematical problem-poems for grades 2–5 are the poetry books by Betsy Franco [8], and Lee Bennett Hopkins [14].

Long out of print, and made obsolete by calculator technology, is the marvellously quaint *Marmaduke Multiply*: *Merry Method of Making Minor Mathematicians* [22]. This 1816 children's classic provides a woodcut illustration and a two-line rhyme for each multiplication from 1×2 to 12×12. Its contents are described on the back cover as: 'it teaches the multiplication table by means of catchy rhymes and engaging woodcuts'.

A different use of poetry, for grades 3–8, may be found, for example, at the Durham District School Board homepage [24]. This use involves creative writing of poetry about mathematics or mathematical concepts by the students. The site contains a brief guideline for teachers wishing to give such assignments in their classes, and an interesting remark by the grade 4/5 (gifted) teacher, Karen Beatty: 'I found that those students who had an excellent understanding of math concepts and vocabulary produced the better poems. This activity would work better at the end of a math unit as a review of the concepts taught rather than the open topic of just math'.

Three sources of poetry appropriate for use in high-school and college mathematics classes are: David Pleacher's *The Handley Math Page*: *Mathematical Poems and Songs* [28], Lawrence Mark Lesser's article in the NCTM journal, The Mathematics Teacher, *Sum of Songs*: *Making Mathematics Less Monotone* [20] and Jacqueline Jensen's Sam Houston State University calculus course poem collection [18]. These sources offer, in addition to mathematical poems meant to be read, lyrics which may be either read or sung to the tune of existing songs. The poems and lyrics on these sites are not mathematical problem-poems, but poems and songs about mathematics, mathematical concepts and the connections between mathematics and other human endeavours. Inclusion of such poems and songs in the curriculum is viewed as an enrichment initiative. Jensen gives a graded assignment requiring students to discover poems with mathematical themes, in order to emphasize the connection between mathematics and other disciplines. Lesser provides as motivation for including such lyrics and songs in mathematics courses the enjoyment students, and teachers, express at the songs themselves, as well as the appreciation of making the unexpected connections between mathematics and songs. The goal is incorporated in the title of Lesser's article – 'less monotone' meaning more interesting and engaging, and as a result – contributing to the enhancement of both learning and mastery of concepts. Lesser also offers a number of classroom enrichment activities for the students which range from discussion of the mathematical concepts appearing in the lyrics to writing lyrics and songs about mathematical concepts.

The most common use of poetry and other arts in college mathematics classes is in general education courses designed for students who intend to major in the humanities. The aim is to reach out to students and develop an appreciation for mathematics, or develop the mathematical thought process, rather than to teach specific material. And indeed many of the poems in Sections 2 and 3 may also be used in this way. In particular, we used the *Lilavati's Swarm* poetry project successfully in a problem-solving course of this kind, restricting our requirement to the finding of solutions by trial and error, and reflection on the emerging patterns. A different approach in the same direction is to dedicate the entire course to the exploration of the connections between mathematics and poetry. This was the approach taken by Marcia Birken and Anne Coon, emeritus professors of Mathematics and English (respectively) at Rochester Institute of Technology. Their 25 year long collaboration in cross-disciplinary research includes the creation and team-teaching of general education courses exploring the connections between mathematics and poetry. Ideas and examples of poetry used in their courses are collected in [3].

The nation-wide education initiative 'writing across the curriculum' (see, for example, [7]) generated a number of recent pedagogical experiments with poetry writing in college mathematics classes. An article by Cheryl and Wayne Patterson [27] reports on an experiment in poetry writing in a quantitative business course involving statistics. In their own words 'we employ poetry assignments to persuade students to think about a unit of material or an exercise they have just completed so that they will be more capable of applying and extending what they learned to new situations'. In other words, poetry writing as a review activity facilitates both 'understanding and integration of subject matter'. Patrick Bahls reports on several poetry writing assignments he gave in precalculus and calculus courses [1]. Bahls required students to write poems connected to mathematics in form or content. His goal was not to review certain material, but rather 'to give students an alternative discourse in which they could explore mathematical ideas'. He reports that such an assignment 'strengthens students' cognitive understanding of mathematical concepts and simultaneously bolsters students' confidence in carrying out mathematical operations'.

These reports, and our own experience with the pedagogical power of poetry projects, leads us to believe that it is worth conducting further experiments with the use of poetry in higher level mathematics courses. Below are two poems that may be used in calculus classes to motivate learning by piquing students' curiosity, enriching their learning experience by highlighting unexpected connections between mathematics and poetry, and placing what is learned in the classroom in a broader historical, artistic and social context.

The first poem, *Calculus* (see page 131), by Sarah Glaz [11,12], deals with the history and the meaning of *The Fundamental Theorem of Calculus*, and the notion of an integral.

This poem may be offered as a handout in class, a gift rather than an assignment. But it is also suitable for designing an enrichment activity based on its content. For example, a possible enrichment activity may take the form mentioned by Lesser [20] of a class discussion of the poem's mathematical content. Another poem suitable for this kind of enrichment activity is *The Derivative Song*, by Tom Lehrer [19], also available online at [33]. Lehrer's poem touches on another fundamental notion of calculus, the definition of the derivative as a limit.

If $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4x^3+x^2-12}{\sqrt{2x^2-9}}$, *then* (see page 131), by Amy Quan Barry [2,12,26], is a lovely poem containing mathematical language and imagery, and an intriguing derivative in its title.

This poem is suitable for designing a 'review of a mathematical unit' project. Specifically, students may be asked to find y, that is, to integrate the derivative in the title. The calculation of this integral is an excellent project for reviewing practically every integration technique learned in a Calculus II course. Depending on the mathematical ability of the students

Calculus

by Sarah Glaz

I tell my students the story of Newton versus Leibniz,
the war of symbols, lasting five generations,
between The Continent and British Isles,
involving deeply hurt sensibilities,
and grievous blows to national pride;
on such weighty issues as publication priority
and working systems of logical notation:
whether the derivative must be denoted by a 'prime',
an apostrophe atop the right hand corner of a function,
evaluated by Newton's fluxions method, $\Delta y/\Delta x$;
or by a formal quotient of differentials $\mathrm{d}y/\mathrm{d}x$,
intimating future possibilities,
terminology that guides the mind.
The genius of both men lies in grasping simplicity
out of the swirl of ideas guarded by Chaos,
becoming channels, through which her light poured clarity
on the relation binding slope of tangent line
to area of planar region lying below a curve,
The Fundamental Theorem of Calculus,
basis of modern mathematics, claims nothing more.

While Leibniz—suave, debonair, philosopher and politician,
published his proof to jubilant cheers of continental followers,
the Isles seethed unnerved,
they knew of Newton's secret files,
locked in deep secret drawers—
for fear of theft and stranger paranoid delusions,
hiding an earlier version of the same result.
The battle escalated to public accusation,
charges of blatant plagiarism,
excommunication from The Royal Math. Society,
a few blackened eyes,
(no duels);
and raged for long after both men were buried,
splitting Isles from Continent, barring unified progress,
till black bile drained and turbulent spirits becalmed.

Calculus—Latin for small stones,
primitive means of calculation; evolving to abaci;
later to principles of enumeration advanced by widespread use
of the Hindu-Arabic numeral system employed to this day,
as practiced by *algebristas*—barbers and bone setters in Medieval Spain;
before Calculus came the Σ (sigma) notion—
sums of infinite yet countable series;
and culminating in addition of uncountable many dimensionless line segments—
the integral $\int$—snake,
first to thirst for knowledge, at any price.

That abstract concepts, applicable—at start,
merely to the unseen unsensed objects: orbits of distant stars,
could generate intense earthly passions,
is inconceivable today;
when Mathematics is considered a dry discipline,
depleted of life sap, devoid of emotion,
alive only in convoluted brain cells of weird scientific minds.

If $\frac{dy}{dx} = \frac{4x^3+x^2-12}{\sqrt{2x^2-9}}$, then,

by Amy Quan Barry

you are standing at the ocean,
in the moon's empirical light
each mercurial wave

like a parabola shifting on its axis,
the sea's dunes differentiated & graphed.
If this, then that. The poet

laughs. She wants to lie
in her own equation, the point slope
like a woman whispering *stay me*

with flagons. What is it to know the absolute value
of negative grace, to calculate
how the heart becomes the empty set

unintersectable, the first & the last?
But enough.
You are standing on the shore,

the parameters like wooden stakes.
Let x be the moon like a notary.
Let y be all things left unsaid.

Let the constant be the gold earth
waiting to envelop what remains,
the sieves of the lungs like two cones.

in class, one may give hints by suggesting some of the following steps:

1. Show that
$$\int \sec^3 t\, dt = \frac{\sec t \tan t + \ln|\sec t + \tan t|}{2} + c$$
2. Show that
$$\int \sec^4 t\, dt = \sec^2 t \tan t - \frac{2}{3}\tan^3 t + c$$
3. Use a trig substitution to show that:
$$\int \frac{4x^3+x^2-12}{\sqrt{2x^2-9}}\, dx = \frac{(8x^2+3x+72)\sqrt{2x^2-9}}{12} - \frac{39\ln\left|(x\sqrt{2}+\sqrt{2x^2-9})/3\right|}{4\sqrt{2}} + c$$
4. Do not get discouraged if this integral takes you three or four pages to calculate.
5. Now calculate this integral using Mathematica. You may do this online at *The Integrator* site [39]. Note that the formula for the integral that you obtain with Mathematica looks different than the formula given in 3. Show that the value of the integral is the same in both cases.

The combination of poetry and integration techniques may yet make both techniques, and poem, memorable.

5. Concluding remarks

Math 1011Q: Introductory College Algebra and Mathematical Modelling is a new course. The number of students who have completed both Math 1011Q and subsequent science courses is still too small for a definitive data analysis. Nevertheless, monitoring progress we see initial signs of success. Classes are very well attended, and students' lively participation in classroom activities shows a lower level of math anxiety than is usually associated with students requiring remedial work. Students' satisfaction, as shown in teaching evaluations, is very high. A preliminary statistical analysis, comparing grades received in Math 1011Q and those received in the next science course (Figure 1), shows that students are doing well in subsequent science courses, and there is a significant correlation (0.4168) between students' performance in Math 1011Q and their performance in the next science course. No numerical data is available for the course that Math 1011Q was designed to replace. The previous course is no longer offered at the University of Connecticut, but the university committee that explored the situation at that time came to the conclusion that after completing the previous course students did not perform well in subsequent science courses. In fact, this conclusion was the driving force behind the request to create Math 1011Q. As we continue teaching this course we intend to collect more evidence, both qualitative and quantitative, on the course's effectiveness, and on the effects of using poetry as a pedagogical tool.

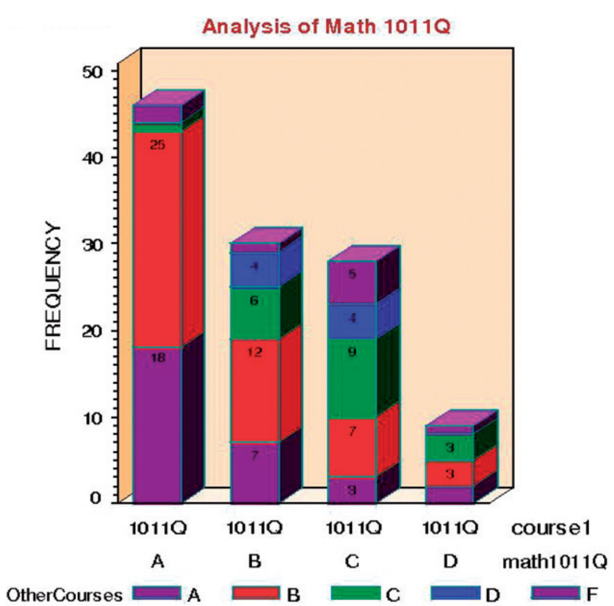

Figure 1. Analysis of grades in Math 1011Q and in the next science course.

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