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Prerequisites: MATH 211, with a grade of C or better

This course begins with the Fundamental Theorem of Calculus and continues with its applications and relation to differential calculus. We will cover: *Chapter 4* (the definite Riemann integral), *Chapter 5* (transcendental functions) and *Chapter 8* (techniques of integration). An assignment syllabus will be provided for each chapter; suggested exercises for acquiring the skills expected on the exams will be provided. Certain exercises will be designated as Assignment exercises; these are to be written up and will be graded, accounting for 15% of your course grade. The remaining 85% of the grade will be from an in-class exam (20%), a project assignment (25%) and the final exam (40%).  

It may help to think of the course in terms of four main ideas:  

I. The definite integral and the Fundamental Theorem of Calculus  
II. Transcendental functions  
III. Inverse Function Theorem  
IV. Techniques of integration  

Course grades will be determined, according to the above weighting, as follows:  

<table>
<thead>
<tr>
<th>Grade</th>
<th>Percentage</th>
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<tbody>
<tr>
<td>A</td>
<td>91+</td>
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<tr>
<td>A−</td>
<td>86−90</td>
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<td>B+</td>
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<td>B</td>
<td>76−80</td>
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<td>B−</td>
<td>71−75</td>
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<td>C+</td>
<td>66−70</td>
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<td>C</td>
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<td>D</td>
<td>45−55</td>
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<tr>
<td>F</td>
<td>&lt;45</td>
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Some important dates:  

2010-04-05 - First day of class  
2010-04-21 - First Exam  
2010-04-23 - CENSUS (Last Day to Drop Without Record of Enrollment)  
2010-04-27 - CAMPUS CLOSED  
2010-05-31 - CAMPUS CLOSED  
2010-06-14 - Last day of class  
2009-06-16 - Final Exam
Notes:

1) Mid-term exam dates are subject to change.

2) Due dates for the graded exercises will be set as we approach the end of each chapter of the text. Assignment must be presented either in an 8\( \frac{1}{2} \) x 11 “blue book” or submitted to me by E-mail as a .pdf file.

3) The only attendance requirement is that you complete the MDTP CR test within the first two weeks of class: Go to mdtp.ucsd.edu, click on On-Line Tests and select the Calculus Readiness Test. Submit the results to me either by E-mail or as a print out. The results do not factor into your course grade in any way. The purpose is to inform you of what review is needed and to inform me of what review skills need to be emphasized. If you cannot access the on-line tests I can arrange for you to take it in hardcopy form at a convenient time.

4) If you are in need of an accommodation for a disability in order to participate in this class, please contact Services to Students with Disabilities at UH-183, (909)537-5238.

5) Excluding very special circumstances, I will not approve petitions to drop the course after April 23. Please refer to the Academic Regulations and Policies section of your current bulletin for information regarding add/drop procedures and consequences of academic dishonesty.

First Exam Study Guide:

1) Antiderivatives/Indefinite Integrals

2) Differential Equations
   a) general solution
   b) specific solution (given initial conditions)
   c) motion in a straight line under constant acceleration

3) Summation Notation
   a) approximating areas
   b) limits of sums and exact areas

4) Riemann Sums and Definite Integrals
   a) constructing a Riemann sum
   b) limits of Riemann sums for continuous functions
   c) using properties of definite integrals (linearity, summability) to evaluate them, particularly from known integrals
   d) area interpretation for non-negative functions

5) Fundamental Theorem of Calculus
   a) \( \int_a^b f(x)dx = F(b) - F(a) \)
   b) MVT for integrals
   c) average value of \( f \) on \([a,b]\)
   d) derivative of \( F(x) = \int_{x_0}^{u(x)} f(t)dt \)
The Suggested Exercises are your guide for the in-class exams. They are a representative sample of techniques required for basic mastery. To reinforce written communication skills the Graded Assignment should be clearly presented with complete explanation of solution steps.

Chapter 4 Suggested Exercises (Larson/Hostetler/Edwards, 9th ed.):

4.1 (pages 255-258):
1 – 7, odd
15 – 49, odd
61, 73, 77, 83

4-2 (pages 267-270):
1 – 13, odd
25, 31, 35, 37, 43
47, 49, 65

4.3 (pages 278-281):
1, 5, 11, 18, 19
31, 39, 43, 57

4.4 (pages 293-296):
5 – 57, odd
61, 67, 75, 81, 87, 89, 101

4.5 (pages 306-310):
1, 3, 5
11 – 39, odd
51, 53, 59
64, 71, 93

First Graded Assignment: Due April 26
Pages 318-320: 18, 22, 46, 64, 74
Complete solutions are required for full credit.
MATH 212-02 Spring/2010: First Exam Study Guide

1) Antiderivatives/Indeﬁnite Integrals
2) Diﬀerential Equations
   a) general solution
   b) speciﬁc solution (given initial conditions)
   c) motion in a straight line under constant acceleration

3) Summation Notation
   a) approximating areas
   b) limits of sums and exact areas

4) Riemann Sums and Deﬁnite Integrals
   a) constructing a Riemann sum
   b) limits of Riemann sums for continuous functions
   c) using properties of deﬁnite integrals (linearity, summability) to evaluate them, particularly from known integrals
   d) area interpretation for non-negative functions

5) Fundamental Theorem of Calculus
   a) \( \int_a^b f(x)dx = F(b) - F(a) \)
   b) MVT for integrals
   c) average value of \( f \) on \([a, b]\)
   d) derivative of \( F(x) = \int_{x_0}^{u(x)} f(t)dt \)
Evaluate the definite integral \( \int_{0}^{4} x^4 \, dx \) by the limit definition as follows:

a) Find \( \Delta x \) for the regular partition with \( n \) subintervals.

\[
\Delta x = \frac{4 - 0}{n} = \frac{4}{n}
\]

b) Find an expression for \( c_i \), the right-hand endpoint of the \( i^{th} \) subinterval.

\[
0 + i \Delta x = \frac{4i}{n}
\]

c) Set up the Riemann sum \( \sum_{i=1}^{n} f(c_i) \Delta x_i \).

\[
\sum_{i=1}^{n} f(c_i) \Delta x_i = \sum_{i=1}^{n} \left( \frac{4i}{n} \right)^4 \frac{4}{n}
\]

d) Write the sum in terms of \( n \) only using the formula \( \sum_{i=1}^{n} i^4 = \frac{1}{30} n (n + 1) (2n + 1) (3n^2 + 3n - 1) \).

\[
\sum_{i=1}^{n} \left( \frac{4i}{n} \right)^4 \frac{4}{n} = \left( \frac{4}{n} \right)^5 \sum_{i=1}^{n} i^4
\]

\[
= \frac{1024}{30n^5} n (n + 1) (2n + 1) (3n^2 + 3n - 1)
\]

\[
= \frac{1024 (n + 1) (2n + 1) (3n^2 + 3n - 1)}{30 n^4}
\]

e) Find the limit as \( n \to \infty \).

\[
\lim_{n \to \infty} \frac{1024 (n + 1) (2n + 1) (3n^2 + 3n - 1)}{30 n^4}
\]

\[
= \frac{1024}{5} = 204.8
\]
Chapter 5 *(Larson/Hostetler/Edwards, 9th ed.):*

The Suggested Exercises are your guide for the in-class exams. They are a representative sample of techniques required for basic mastery.

5.1 *(pages 331-333):*
- 7 – 18
- 29, 31, 33, 35
- 39 – 75, odd
- 83, 87, 89, 95, 101

5.2 *(pages 340-342):*
- 1 – 39, odd
- 53, 59, 69, 75, 77

5.3 *(pages 349-351):*
- 9 – 12
- 23 – 29, odd
- 41, 45, 51, 61, 71, 83

5.4 *(pages 358-361):*
- 17, 21, 25 – 28, 31, 37
- 39 – 75, odd
- 83, 87, 97, 105, 107, 109, 125, 129

5.5 *(pages 368-372):*
- 39, 41, 47, 49, 59, 71, 83

5.6 *(pages 379-381):*
- 5, 7, 9, 17, 21 – 33, odd
- 41 – 51, odd
- 61, 63, 75, 81, 99

5.7 *(pages 387-389):*
- 1, 3, 5, 7, 25, 29, 31
- 39, 51, 61, 75

5.8 *(pages 398-400):*
- 7 – 39, odd
- 43, 49, 61, 63, 85, 89, 107

Second Graded Assignment. Complete solutions required for full credit. Due: May 19.

*(Numbers in parentheses are from 8th edition.)*
- Page 359: 87 *(Page 357: 73)*
- Page 361: 151 *(Page 357: 75)*
- Page 404: 10, 11, 13 *(Page 402: 8, 9, 10)*

Project Assignment. Do one of the following. Solution must be in print form. Due: June 2.
- Page 400: 114 *(Page 398: 94)*
- Page 403: 6 *(Page 401: 6)
A function defined by a definite integral:

\[ F(x) = \int_{0}^{x} \sin t \, dt \]

\[ \lim_{x \to \infty} F(x) = \frac{1}{2} \pi \]

\[ F'(x) = \frac{\sin x}{x} \]

\[ F''(x) = \frac{1}{x} \cos x - \frac{1}{x^2} \sin x \]

\[ F(x) = \int_{0}^{x} \frac{\sin t}{t} \, dt \] and its derivative \( f(x) = \frac{\sin x}{x} \).
Schedule of remaining topics:

May 10: Differentiation of Inverse Trigonometric Functions

May 12: Integration of Inverse Trigonometric Functions

May 17: Applications of Hyperbolic Functions

May 19: Integration By Parts; Chapter 5 Assignment Due

May 24: Trigonometric Integrands and Related Substitutions

May 26: Partial Fractions

June 2: Partial Fractions (continued); Project Due

June 7: L'Hôpital's Rule

June 9: Improper Riemann Integrals

June 14: Review; Chapter 8 Assignment Due

June 16: Final Exam
Vertical motion with air resistance:

If \( v_0 = 0 \), 
\( s(t) = s_0 - 16t^2 \) without taking air resistance into account.

Assume 
\[ v'(t) = -32 + kv^2, \quad k > 0 \]

Then 
\[ \frac{1}{32} \int \frac{1}{1 - \left( \frac{k}{32} v \right)^2} dv = -\int dt \]
\[ \frac{1}{\sqrt{32k}} \int \frac{1}{1 - u^2} du = -t, \quad u = \frac{1}{\sqrt{32}} v \]

Thus, 
\[ \frac{1}{\sqrt{32k}} \tanh^{-1} \left( \sqrt{\frac{k}{32}} v \right) = -t + C, \quad \text{where } C = \frac{1}{\sqrt{32k}} \tanh^{-1} \left( \sqrt{\frac{k}{32}} v_0 \right) = 0 \]

since \( v_0 = 0 \).

It follows that 
\[ v(t) = -\sqrt{\frac{32}{k}} \tanh \left( \sqrt{32kt} \right) \]
\[ \lim_{t \to \infty} v(t) = -\sqrt{\frac{32}{k}} \]

For example, the terminal velocity for a skydiver is about 200 ft/s. Find \( k \).

Further
\[ s(t) = -\sqrt{\frac{32}{k}} \int \tanh \left( \sqrt{32kt} \right) dt \]
\[ = -\frac{1}{k} \ln \cosh \left( \sqrt{32kt} \right) + s_0 \]

For example, if 
\( \quad k = .001 \)
\( \quad s_0 = 10000 \)
then \( s(t) = 0 \) when 
\( t \approx 59.777 \)

**Note:** With \( k = 0 \), 
\( s_0 - 16t^2 = 0 \), when \( t = 25 \).
Gudermannian function:

\[ G(x) = \int_0^x \text{sech } t \, dt \]

\[ G'(x) = \text{sech } x \]
\[ G''(x) = \text{sech } x \tanh x \]

\[ g(x) = \arctan(\sinh x) \]
\[ g'(x) = \frac{\cosh x}{\sinh^2 x + 1} = \text{sech } x \quad \text{and} \quad g(0) = G(0) \quad \text{so} \quad g = G \]

\[ h(x) = \arcsin(\tanh x) \]
\[ h'(x) = \sqrt{1 - \tanh^2 x} = \text{sech } x \quad \text{and} \quad h(0) = G(0) \quad \text{so} \quad h = G \]
MATH 212-02: Spring 2010

The Suggested Exercises are your guide for the in-class exams. They are a representative sample of techniques required for basic mastery.

To reinforce written communication skills the Graded Assignment should be clearly presented with complete explanation of solution steps.

Chapter 8 Suggested Exercises (Larson/Hostetler/Edwards, 9th ed.):

8.1 (pages 524-526):
1, 13, 23, 35, 41, 47, 49, 59, 77

8.2 (pages 533-535):
7, 11, 15, 19, 27, 33, 35, 43
73, 83, 107

8.3 (pages 542-544):
5, 7, 9, 17
25, 43, 51, 71, 93

8.4 (pages 551-553):
5, 9, 15, 17, 19, 27, 35, 47, 53

8.5 (pages 561-562):
9, 11, 13, 25, 31, 41

8.6 (pages 567-568):
63

8.7 (pages 576-579):
9, 13, 15, 21, 35
93, 95

8.8 (pages 587-590):
9, 11, 19, 27, 33, 77, 91, 107

Third Graded Assignment. Complete solutions required for full credit. Due: June 14.

(Numbers in parentheses are from 8th edition.)
Page 591: 22, 40 (Page 589: 20, 38)
Page 592: 54, 70, 78 (Page 590: 52, 68, 76)
An application of partial fractions integration to probability:

Let \( f(x) = \frac{\sqrt{2}}{\pi} \frac{1}{x^4+1} \)

\[ y = f(x); \text{ dashed curve is the standard normal density function} \]
\[ y = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \]

which could be a probability density function for a certain random variable. This means that the probability of this variable falling with the interval \([a, b]\) is given by

\[ \int_a^b f(x)dx \]

Using the FTC we can evaluate these probabilities if we find an antiderivative \(F\) for \(f\). First, decompose \(\frac{1}{x^4+1}\) into partial fractions, which requires the factorization

\[ x^4 + 1 = (x^2 + \sqrt{2}x + 1)\left(x^2 - \sqrt{2}x + 1\right) \]

and so

\[ \frac{1}{x^4+1} = \frac{Ax + B}{x^2 + \sqrt{2}x + 1} + \frac{Cx + D}{x^2 - \sqrt{2}x + 1} \]

which yields the identity

\[ x^3(A + C) + x^2(B + D - A\sqrt{2} + C\sqrt{2}) + x(A + C - B\sqrt{2} + D\sqrt{2}) + (B + D) = 1 \]
and thus the system
\begin{align*}
A + C &= 0 \\
B + D - A\sqrt{2} + C\sqrt{2} &= 0 \\
A + C - B\sqrt{2} + D\sqrt{2} &= 0 \\
B + D &= 1
\end{align*}

with the unique solution
\begin{align*}
A &= \frac{\sqrt{2}}{4} = -C \\
B &= \frac{1}{2} = D
\end{align*}

Then \( \frac{2}{\sqrt{2}} F(x) = \int \frac{\sqrt{2}x^2 + 1}{x^2 + \sqrt{2}x + 1} \, dx + \int \frac{-\sqrt{2}x^2 - 1}{x^2 - \sqrt{2}x + 1} \, dx = u(x) + v(x) \), and we calculate \( u(x) \), \( v(x) \) after completing the squares in the denominators:
\begin{align*}
x^2 + \sqrt{2}x + 1 &= \left( x + \frac{1}{\sqrt{2}} \right)^2 + \frac{1}{2} \\
x^2 - \sqrt{2}x + 1 &= \left( x - \frac{1}{\sqrt{2}} \right)^2 + \frac{1}{2}
\end{align*}

For \( u(x) \), let \( w = x + \frac{1}{\sqrt{2}} \). Then \( dx = dw \) and
\[ u(x) = \frac{1}{4} \int \frac{\sqrt{2}w + 1}{w^2 + \left( \frac{1}{\sqrt{2}} \right)^2} \, dw \]

For \( v(x) \), let \( w = x - \frac{1}{\sqrt{2}} \). Then \( dx = dw \) and
\[ v(x) = \frac{1}{4} \int \frac{1 - \sqrt{2}w}{w^2 + \left( \frac{1}{\sqrt{2}} \right)^2} \, dw \]

It follows that
\begin{align*}
u(x) &= \frac{1}{4} \sqrt{2} \arctan \left( x\sqrt{2} + 1 \right) + \frac{1}{8} \sqrt{2} \ln \left( x^2 + x\sqrt{2} + 1 \right) \\
v(x) &= \frac{1}{4} \sqrt{2} \arctan \left( x\sqrt{2} - 1 \right) - \frac{1}{8} \sqrt{2} \ln \left( x^2 - x\sqrt{2} + 1 \right)
\end{align*}

and so
\[ F(x) = \frac{1}{2\pi} \left( \ln \sqrt{\frac{x^2 + x\sqrt{2} + 1}{x^2 - x\sqrt{2} + 1}} + \arctan \left( x\sqrt{2} + 1 \right) + \arctan \left( x\sqrt{2} - 1 \right) \right) + C \]

If we let \( C = \frac{1}{2} \) then \( \lim_{x \to \infty} F(x) = 1 \), and \( F \) becomes a cumulative distribution function for the probability density function \( f \):
As an example, suppose the random variable measures the distance east (negative) or west (positive) of a station on a railroad track from which a hazard report might come in. What is the probability that the report will originate within 1 mile of the station? This probability is given by the definite integral
\[
\int_{-1}^{1} f(x)\,dx = F(1) - F(-1)
\]
\[
= \frac{1}{2\pi} \ln \left(2\sqrt{2} + 3\right) + \frac{1}{2}
\]
\[
\approx 0.78055
\]
so there is a 78% probability that the report will occur within 1 mile of the station.

**Exercise:** The *standard deviation* $\sigma$ of this random variable is defined by
\[
\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx.
\]

Find $\sigma$ and calculate the probability of a measurement falling within 1 standard deviation on either side of $x = 0$. 

14
Integration by substitution
Indefinite integrals $\int f(g(x))g'(x)\,dx = \int f(u)\,du = F(u) + C$
Definite integrals $\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$

Natural log function: $\ln x = \int_1^x \frac{1}{t}\,dt, x > 0$
- Domain, range, base $e$
  \[ \frac{d}{dx} \ln u(x) = \frac{1}{u(x)} u'(x) \]
- Logarithmic differentiation
  \[ \text{Antiderivatives: } \frac{d}{dx} \ln |u| = \frac{u'}{u} \text{ so } \int \frac{1}{u} du = \ln |u| + C \]
- Integrands involving trigonometric functions

Inverse Function Theorem
  If $f$ has an inverse function $g$ and $f'(g(x)) \neq 0$ then $g'(x) = \frac{1}{f'(g(x))}$.

Exponential functions
  \[ f^{-1}(x) = e^x \text{ where } f(x) = \ln x \]
  \[ \frac{d}{dx} e^u = e^u \frac{du}{dx} \]
  \[ a^x = e^{x \ln a}, a > 0 \]
  \[ \log_a x = \frac{1}{\ln a} \ln x \]
  \[ \text{Exponential growth and decay} \]

Inverse Trigonometric Functions
  \[ \text{Domains and ranges} \]
  \[ \text{Derivatives} \]

Hyperbolic functions.
  \[ \text{Domains and ranges} \]
  \[ \text{Derivatives and antiderivatives} \]
  \[ \text{Inverse functions} \]

Integration by parts.
  \[ \text{Recognizing } u \text{ and } dv \]
  \[ \text{Multiple step applications} \]

Trigonometric integrals and substitutions.
  \[ \text{Integrands with powers of sin, cos, tan and sec as factors} \]
  \[ \text{Integrands involving } \sqrt{a^2 - u^2}, \sqrt{u^2 - a^2}, \sqrt{u^2 + a^2} \]
Partial fractions.
Decomposing rational functions into terms with powers of linear and quadratic polynomials in the denominator

L’Hôpital’s Rule.
Limits resulting in the indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$
Reduction of other indeterminate forms to these types

Improper integrals.
Determining convergence of definite integrals with infinite limits of integration and/or infinite discontinuities in the interval