4-2 Binomial Distributions

Requirements of Binomial Probability Distributions

1) The experiment has a fixed number of trials \( n \), where each trial is independent of the other trials.

2) There are only two possible outcomes of interest for each trial. Each outcome can be classified as (Success or Failure), (Yes or No), (Favorable or Unfavorable).

3) The probability of success is the same for each trial in all trials.

4) The random variable \( x \) counts the number of successful trials.

Notation for Binomial Probability Distributions:

S or F (success or failure)

\[ P(S) = p \quad (p = \text{probability of a success}) \]

\[ P(F) = 1 - p = q \quad (q = \text{probability of a failure}) \]

\( n \): the fixed number of trials.

\( x \): the number of successes among \( n \) trials \((x = 0, 1, 2, \ldots, n)\).

Binomial Probability Formula:

\[ P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)! x!} \cdot p^x \cdot q^{n-x}, \quad x = 0, 1, 2, \ldots, n. \]

Mean, Variance, and Standard Deviation

Mean: \( \mu = n \cdot p \)

Variance: \( \sigma^2 = n \cdot p \cdot q \)

Standard Deviation: \( \sigma = \sqrt{n \cdot p \cdot q} \)
Binomial Probability Distributions

Find the probability for each value of $X$.

**Ex.** A coin is tossed three times. Find the probability of getting exactly two heads. (50%)

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<th>Value of $X = x$</th>
<th>Probability $P(X = x)$</th>
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**Ex.** A survey from a research found that 30% of teenagers have part-time jobs. If four teenagers are selected at random find the probability that at least one of them will have part-time jobs.

1) At least one ⇒ Find the individual probability for 1, 2, 3, and 4 and add them to get the total probability.

2) At least one ⇒ $1 - P$(None of them has a part-time job)

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Mean, Variance, and Standard deviation for Binomial Probability Distributions

\[ n = 5, \ p = 0.3, \ \text{and} \ q = 0.7, \]

Mean: \[ \mu = n \cdot p \Rightarrow \mu = 5 \cdot 0.3 = 1.5 \]

Variance: \[ \sigma^2 = n \cdot p \cdot q \Rightarrow \sigma^2 = 5 \cdot 0.3 \cdot 0.7 = 1.05 \]

Standard Deviation: \[ \sigma = \sqrt{n \cdot p \cdot q} \]
\[ \sigma = \sqrt{1.05} = 1.02469 \approx 1.025 \]

Ex.

McDonald’s has a 95% recognition rate. A special focus group consists of 12 randomly selected adults. For such a group, find the mean and standard deviation.

Range Rule of Thumb

Maximum usual value: \[ \mu + 2\sigma \]

Minimum usual value: \[ \mu - 2\sigma \]

\[ \mu - 2\sigma \leq \text{usual value} \leq \mu + 2\sigma \]

Poisson Probability Distribution:

For data that represent the number of occurrence of a specific event in a given unit of time or space.

Example

\[ \cdot \text{The number of customer arrivals at a cashier during a given minute.} \]

\[ \cdot \text{The number of calls received by a switchboard during a given period of time.} \]

\[ \cdot \text{The number of machine breakdowns during a given day.} \]

\[ P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!} \]