1. (15 pts.) Of a company’s personnel, seven work in design, 14 in manufacturing, four in testing, five in sales, two in accounting, and three in marketing. A committee of six people is to be formed to meet with upper management. In how many ways can the committee be formed if there must be at least four members from the manufacturing department?

Design: 7
Manufacturing: 14
Testing: 4
Sales: 5
Accounting: 2
Marketing: 3

35 total

Committee
6 people

Choose 4 from manufacturing
\[ C(14, 4) \cdot C(21, 2) \]
Choose 5 from manufacturing
\[ + C(14, 5) \cdot C(21, 1) \]
Choose 6 from manufacturing
\[ + C(14, 6) \]

Answer: 255, 255

2. (15 pts.) How many arrangements of the letters a, b, c, d, e, f begin and end in a vowel?

\[ \frac{2 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1}{a \text{ or } e} = 48 \]

Pick first letter: 2
Pick last letter: 1
Arrange remaining: 4!

Answer: 48
3. (15 pts.) How many ways are there to arrange the letters in CHIHUAHUA?

\[
\frac{C(9,3) \cdot C(6,2) \cdot C(4,2) \cdot C(2,1) \cdot C(1,1)}{\uparrow \uparrow \uparrow \uparrow \uparrow}
\]
place H's \hspace{1cm} place A's \hspace{1cm} place U's \hspace{1cm} place C \hspace{1cm} place I

\[= 84 \cdot 15 \cdot 6 \cdot 2 \cdot 1 = 15120 \]

Answer: 15,120

4. (15 pts.) A bag of marbles contains ten identical green marbles, eight identical red marbles, nine identical yellow marbles, and eight identical blue marbles. How many different (unordered) selections of six marbles from the bag are there?

Green: 10
Red: 8
Yellow: 9
Blue: 8

\[\begin{array}{cccc}
\times & \times & \times & \times \\
\end{array}\]

Green \hspace{1cm} Red \hspace{1cm} Yellow \hspace{1cm} Blue

Arrange 6 x's and 3 lines:

\[C(9,3) = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} = 3 \cdot 4 \cdot 7 = 84\]

Answer: 84
5. (15 pts.) Evaluate the sum

\[ 1 + 2 \binom{n}{1} + \cdots + (k+1) \binom{n}{k} + \cdots + (n+1) \binom{n}{n} \] (1)

by breaking this sum into two sums and using identities. Then use your result to calculate

\[ 1 + 2 \binom{50}{1} + 3 \binom{50}{2} + 4 \binom{50}{3} + \cdots + 51 \binom{50}{50}. \] (2)

\[
(1) = \sum_{k=0}^{n} (k+1) \binom{n}{k} = \sum_{k=0}^{n} k \binom{n}{k} + \sum_{k=0}^{n} \binom{n}{k}
\]

\[
= \sum_{k=1}^{n} k \binom{n}{k} + \sum_{k=0}^{n} \binom{n}{k} = \sum_{k=1}^{n} n \binom{n-1}{k-1} + 2^n
\]

\[
= n \sum_{k=1}^{n} \binom{n-1}{k-1} + 2^n = n \sum_{k=0}^{n-1} \binom{n-1}{k} + 2^n = n \cdot 2^{n-1} + 2^n
\]

\[
= 2^{n-1} \cdot (n+2)
\]

\[
(2) = 2^{50-1} \cdot (50+2) = 2^{49} \cdot (52)
\]

Answer to (1): \[2^{n-1} \cdot (n+2)\]  
Answer to (2): \[52 \cdot 2^{49} \times 2.917 \times 10^{11}\]
6. (15 pts.) Let \(a, b, c, d, e\) and \(f\) all be integers greater than or equal to 0. How many ways are there to assign values to each of these variables so that \(a + b + c + d + e + f = 30\)? (Note that the values assigned need not be distinct, so for example, \(a = 0, b = 12, c = 0, d = 12, e = 2, f = 4\) is one such assignment.)

\[
\begin{array}{ccccccc}
& & & \text{x} & & & \\
& & & \text{x} & \text{x} & \text{x} & \text{x} & \text{x} \\
& & a & b & c & d & e & f \\
\end{array}
\]

Place 30 x's and 5 lines into 35 spaces.

\[
\binom{35}{5} = 324,632
\]

Answer: \(324,632\)

7. (10 pts.) In the game of SET, suppose we restrict our attention to only the red cards in the deck. Among these cards, how many sets are there where exactly one of the cards in the set is a single red diamond?

There are 3 \(\times\) 3 \(\times\) 3 = 27 red cards.

\[
\begin{align*}
\text{2 ways to reorder the two cards} & & \text{24 choices} \quad 1 \text{ choice} = \frac{24}{2} = 12 \\
\text{or} & & \text{24} \quad 1 = \frac{24}{2} = 12 \\
\text{or} & & \text{24} \quad 1 = \frac{24}{2} = 12
\end{align*}
\]

Steps:

1. Pick a diamond 3 choices.
2. Pick a second card 24 choices.
3. Pick a 3rd card 1 choice.

We've double counted steps 2 and 3.

Answer: \(360\)