Tennis:

Set: Best 2 out of 3 sets wins.

(*) To win a set, you have to win 6 games, and you have to win by 2.

(**) To win a game, you have to win 4 points, and you have to win by 2.

Can you think of a game that consists of 8 points?

A, A, B, A, B, B, A, A

Can you think of a game that consists of 7 points? No

Theorem: Michael's Conjecture: Any game that consists of more than 5 points must consist of an even number of points.
Proof: A finished game must consist of at least 4 points. At six points, either the game is finished, or it is tied. If it is tied, we need to play at least 2 more points to finish the game. After 2 points, either the game is finished, or there is once again a tie. Thus, for the game to finish, we must have played

\[ 6 + 2 + 2 + 2 + \cdots + 2 = 6 + 2 \cdot k \]

\[ k \]

points, where \( k \) is a positive integer.

But \[ 6 + 2 \cdot k = 2 \cdot (3 + k) \]. Since \( 3 + k \) is an integer, this is even. \( \Box \)
Josephina Game

40 0. 2
38 0. 3
36 0. 4
34 0. 5

Simpler Problem:
4 people, every 3rd person (start counting at person 1)
1.2: Number Puzzles and Sequences

What is a sequence?

- A list of numbers (in order) that follow a pattern.

A function is something we have used to describe a pattern.

Ex. \( f(x) = 2x + 1 \) \( x \) is a real number.

\[ a_n = f(n) = 2n + 1, \quad n \text{ is an integer} \]

\[ a_n = 2n + 1, n \geq 1 \]

3, 5, 7, 9, 11, ...
3, 5, 7, 11, 13, ...

Ways to describe this sequence:

- \( a_n = 2n + 1, \ n \geq 1 \)
- \( a_n = 2n - 1, \ n \geq 2 \)
- \( a_n = 2n + 3, \ n \geq 0 \)

\( a_0 = 3, \ a_1 = 5, \ a_2 = 7, \ldots \)

- Start at 3 and keep adding 2
  \( a_n = a_{n-1} + 2, \ n \geq 2 \)

Recursive definition of a sequence
Ex. Here's a sequence:

0, 1, 3, 7, 15, 31, 63, ...

Closed Formula:

\[ a_n = 2^n - 1, \quad n \geq 0 \]

Recursive Formula:

\[ a_n = 2 \cdot a_{n-1} + 1, \quad n \geq 2 \]

\[ a_1 = 0 \]