1. Let $F \subseteq B \subseteq E$ be a tower of fields. Prove that $\text{Gal}(E/B)$ is a subgroup of $\text{Gal}(E/F)$.

2. Let $F \subseteq B \subseteq E$ be a tower of fields with $B/F$ the splitting field of some polynomial $f(x) \in F[x]$ and $E/F$ the splitting field of some $g(x) \in F[x]$. Define $\psi : \text{Gal}(E/F) \rightarrow \text{Gal}(B/F)$ by $\sigma \mapsto \sigma|B$. Prove that $\psi$ is a homomorphism and that $\text{Ker } \psi = \text{Gal}(E/B)$.

3. Let $E/F$ be a field extension and $G = \text{Gal}(E/F)$. Then prove that $E^G$ is a subfield of $E$ and $F \subseteq E^G \subseteq E$.

4. Let $E/F$ be a field extension and let and $G = \text{Gal}(E/F)$. If $H_1 \leq H_2 \leq G$, then prove that $E^{H_2} \subseteq E^{H_1}$.

5. If $E/F$ is a Galois extension and $F \subseteq B \subseteq E$, then prove that $E/B$ is a Galois extension.

6. If $E/F$ is a Galois extension and $B$ is an intermediate field with $B/F$ a Galois extension then prove that $B$ has no conjugates other than $B$ itself.

7. Let $E = \mathbb{Q}(\sqrt{2}, \sqrt{3}, i)$.
   
   (a) Construct the subfield lattice for $E$ over $\mathbb{Q}$.
   
   (b) For each $B$ such that $\mathbb{Q} \subseteq B \subseteq E$, calculate $|\text{Gal}(E/B)|$.
   
   (c) Prove $\text{Gal}(E/\mathbb{Q}) \cong (\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2)$.

8. Let $E$ be a Galois extension of a field $F$ such that $\text{Gal}(E/F)$ is abelian. Show that for any intermediate field $F \subseteq B \subseteq E$, $B$ is a Galois extension of $F$.

9. Let $E$ be a Galois extension of a field $F$ such that $\text{Gal}(E/F)$ is cyclic. Show that for any intermediate field $F \subseteq B \subseteq E$, $B$ is a Galois extension of $F$ and both the groups $\text{Gal}(E/B)$ and $\text{Gal}(B/F)$ are cyclic.