1. For each of the following sequences, prove, using an \( \epsilon - n_0 \) argument that the sequence converges to the given limit \( a \); that is, given \( \epsilon > 0 \), determine \( n_0 \) such that \( |a_n - a| < \epsilon \ \forall n \geq n_0 \).

(a) \( \left\{ \frac{6n-2}{5n-7} \right\} \), \( a = \frac{6}{5} \).

(b) \( \left\{ \frac{8n}{7n+3} \right\} \), \( a = 0 \).

(c) \( \left\{ \frac{\sin n}{n} \right\} \), \( a = 0 \).

(d) \( \left\{ \frac{2n+5}{6n-3} \right\} \), \( a = \frac{1}{3} \).

(e) \( \left\{ 1 - \frac{(-1)^n}{n} \right\} \), \( a = 1 \).

(f) \( \left\{ \frac{(-1)^n}{n+1} \right\} \), \( a = 0 \).

(g) \( \left\{ n\sqrt{1 + \frac{1}{n} - 1} \right\} \), \( a = \frac{1}{2} \).

2. Theorem 2.1.10: (a) If a sequence \( \{a_n\} \) converges then its limit is unique.
(b) If a sequence \( \{a_n\} \) converges then \( \{a_n\} \) is bounded.

3. Theorem 2.2.1 If \( \{a_n\} \) and \( \{b_n\} \) are convergent with \( \lim_{n \to \infty} a_n = a \) and \( \lim_{n \to \infty} b_n = b \), then
(a) \( \lim_{n \to \infty} (a_n + b_n) = a + b \)
(b) \( \lim_{n \to \infty} (a_nb_n) = ab \)
(c) Furthermore, if \( a \neq 0 \) and \( a_n \neq 0 \ \forall n \in \mathbb{N} \), then \( \lim_{n \to \infty} \frac{b_n}{a_n} = \frac{b}{a} \)

4. Corollary 2.2.2: If \( \{a_n\} \) is convergent with \( \lim_{n \to \infty} a_n = a \) and \( c \in \mathbb{R} \), then
(a) \( \lim_{n \to \infty} (a_n + c) = a + c \)
(b) \( \lim_{n \to \infty} (a_nc) = ac \)

5. Theorem 2.2.3: Let \( \{a_n\} \) and \( \{b_n\} \) be sequences of real numbers. If \( \{b_n\} \) is bounded and \( \lim_{n \to \infty} a_n = 0 \), then \( \lim_{n \to \infty} a_nb_n = 0 \).

6. Theorem 2.2.4: Suppose \( \{a_n\}, \{b_n\}, \) and \( \{c_n\} \) are sequences for which \( \exists n_0 \in \mathbb{N} \) such that \( a_n \leq b_n \leq c_n \ \forall n \geq n_0 \). If \( \{a_n\} \) and \( \{c_n\} \) converge to \( L \), then \( \{b_n\} \) converges to \( L \).

7. Let \( \{a_n\} \) be a sequence in \( \mathbb{R} \) with \( \lim_{n \to \infty} a_n = a \). Prove that \( \lim_{n \to \infty} (a_n)^2 = a^2 \)

8. Let \( \{a_n\} \) be a sequence in \( \mathbb{R} \) with \( \lim_{n \to \infty} a_n = a \). Prove that \( \lim_{n \to \infty} (a_n)^3 = a^3 \).

9. If \( \{a_n\} \) converges to \( a \), then \( \{|a_n|\} \) converges to \(|a|\). The converse is false.

10. Let \( \{a_n\} \) and \( \{b_n\} \) be sequences of real numbers.
(a) If \( \{a_n\} \) and \( \{a_n + b_n\} \) both converge, prove that the sequence \( \{b_n\} \) converges.
(b) Suppose \( b_n \neq 0 \ \forall n \in \mathbb{N} \). If \( \{b_n\} \) and \( \left\{ \frac{a_n}{b_n} \right\} \) both converge, prove that the sequence \( \{a_n\} \) also converges.