A-1 Find polynomials \( f(x), g(x), \) and \( h(x), \) if they exist, such that for all \( x, \)
\[
|f(x)| - |g(x)| + h(x) = \begin{cases} 
-1 & \text{if } x < -1 \\
3x + 2 & \text{if } -1 \leq x \leq 0 \\
-2x + 2 & \text{if } x > 0.
\end{cases}
\]

A-4 Sum the series

\[
\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2n}{3^m(n3^m + m3^n)}.
\]

B-2 Let \( P(x) \) be a polynomial of degree \( n \) such that \( P(x) = Q(x)P''(x), \) where \( Q(x) \) is a quadratic polynomial and \( P''(x) \) is the second derivative of \( P(x). \) Show that if \( P(x) \) has at least two distinct roots then it must have \( n \) distinct roots.

B-5 For an integer \( n \geq 3, \) let \( \theta = 2\pi/n. \) Evaluate the determinant of the \( n \times n \) matrix \( I+A, \) where \( I \) is the \( n \times n \) identity matrix and \( A = (a_{jk}) \) has entries \( a_{jk} = \cos(j\theta+k\theta) \) for all \( j, k. \)

B-6 Let \( S \) be a finite set of integers, each greater than 1. Suppose that for each integer \( n \) there is some \( s \in S \) such that \( \gcd(s, n) = 1 \) or \( \gcd(s, n) = s. \) Show that there exist \( s, t \in S \) such that \( \gcd(s, t) \) is prime.