Problem of the Month, March 2008

Please turn all solutions into Dr. Dunn’s office, JB 322. You may slide your solutions under his door as well. Most elegant solution wins a $10 gift certificate to the bookstore! Solutions will be accepted anytime during the month of March, 2008. This month’s problem is dedicated to Dr. Dunn’s brand new niece, Evie Mae Dunn, who was born on March 4th, 2008. Good luck!

Draw a square on your paper with side length 1. Find the midpoints of each of the 4 sides, and connect these points in such a way to make another square. Then find the midpoints of each of the sides of this new square and connect them to make yet another square. Continue this process to form an infinite sequence of squares. Now consider the right triangles embedded in this square. Shade in the triangles which have their legs parallel to the sides of the original square, and find the area which is shaded. (In the pictures below, we demonstrate the object described being constructed, and illustrate a partial shading. Keep in mind that the embedded squares continue indefinitely!)