Problem of the Month, January 2009

Please turn all solutions into Dr. Dunn’s office, JB 322. You may slide your solutions under his door as well. Most elegant solution wins a $10 gift certificate to the bookstore! Solutions will be accepted anytime during the month of January, 2009. Good luck!

Let $S_1$ be a square of side length 2. Inscribe a circle of maximal area into $S_1$, and inside of that circle inscribe a square $S_2$ of maximal area. Inscribe a circle of maximal area inside $S_2$, and then inscribe a square $S_3$ inside of that circle. Continuing this process, we get a sequence of (nested) squares $S_1, S_2, \ldots$. Let $A(S_k)$ be the area of the square $S_k$. Compute $\sum_{k=1}^{\infty} A(S_k)$. 