Problem of the Month, February 2009

Please turn all solutions into Dr. Dunn’s office, JB 322. You may slide your solutions under his door as well. Most elegant solution of this month’s problem wins TWO $10 gift certificates to the bookstore! Solutions will be accepted anytime during the month of February, 2009. Good luck!

Let $P$ and $Q$ be points on the hollow sphere of radius one in $\mathbb{R}^3$. There exists a unique great circle on the sphere which connects the points. Please find a formula that expresses the (shortest) distance between these points along the great circle connecting them. Your formula should not have any unevaluated integrals in it. (You may find it helpful to know what the actual definition of a radian is!)