Hi everyone. Please work as efficiently as possible. I'm going to go get a fax from my future self.

1. Number 2: There are 7 nonidentity elements of order 2. Each of these elements, along with the identity of the group, form a subgroup of order 2, and each of these subgroups are distinct.

2. Number 5: We prove that \( \mathbb{Z} \oplus \mathbb{Z} \) is not cyclic, which shows that \((\text{cyclic}) \oplus (\text{cyclic})\) need not be cyclic. Suppose that \( \mathbb{Z} \oplus \mathbb{Z} = \langle (a, b) \rangle \). Then \((0, b) \in \langle (a, b) \rangle \), and so there exists a \( k \) so that \((0, b) = k(a, b) = (ka, kb)\), and so \( 0 = ka \) and \( b = kb \). If \( b \neq 0 \), then \( k = 1 \). And if \( k = 1 \), then \( a = 0 \). So in either case, either \( a \) or \( b \) is zero. If \( a = 0 \), then the element \((1, 0) \notin \langle (0, b) \rangle \), and a similar contradiction holds if \( b = 0 \). Thus \( \mathbb{Z} \oplus \mathbb{Z} \) is not cyclic.

3. Number 12: Suppose that \( R \) is the cyclic subgroup generated by the rotations in \( D_n \), and that \( F \) is any subgroup of order 2. Since \( R \) is cyclic, it is abelian, and since the order of \( F \) is prime, then \( F \) must be cyclic, and hence abelian. Then \( R \oplus F \) is abelian, and thus it must not be isomorphic to the nonabelian group \( D_n \).
4. Number 46: We map \( \varphi(ax^2 + bx + c) = (a, b, c) \). This is clearly well defined, and we check that it is an isomorphism. First, we observe

\[
\varphi((ax^2 + bx + c) + (dx^2 + ex + f)) = \varphi((a + d)x^2 + (b + e)x + (c + f)) = (a + d, b + e, c + f) = (a, b, c) + (d, e, f) = \varphi(ax^2 + bx + c) + \varphi(dx^2 + ex + f).
\]

So \( \varphi \) is a homomorphism. We can see that \((a, b, c) = \varphi(ax^2 + bx + c) \) for any \((a, b, c) \in \mathbb{Z}_3 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3\), and so \( \varphi \) is onto. Finally, if \( \varphi(ax^2 + bx + c) = (0, 0, 0) \), then it follows that \( a = b = c = 0 \), and so as the kernel of \( \varphi \) is trivial, this homomorphism is one-to-one as well. Thus, it is an isomorphism.

The generalization comes from taking any abelian groups \( G_0, \ldots, G_n \), and arranging them (symbolically) into a polynomial group as \( G = G_nx^n + \cdots + G_1x + G_0 = \{g_nx^n + \cdots + g_1x + g_0 | g_i \in G_i \} \), and that this group \( G \cong G_0 \oplus \cdots \oplus G_n \). Usually this is done in this setting where the \( G_i \) are all the same group, but sometimes not. ROCK ON!