Hi everyone. My name is John, one of the many raccoons that steals cat food at night from Corey’s residence. I guess Corey finally had had enough, and decided to trap my family and I, one by one if he has to, and ferry us out into the mountains. Before I left, though, I wanted to write the solutions to the first homework assignment as a way of thanking Corey for letting me feast on his food. Rock on!

1. # 1 from the handout. Corey graded this question, but found nobody who appeared to have any trouble with it. So instead of making the huge tables, Corey told me it was okay to encourage you all to just talk to him if you need more information about this one.

2. # 2 from the handout. It is true that if $a, b \in \mathbb{Z}_3$ are nonzero elements, then $ab \neq 0 \in \mathbb{Z}_3$ as well. This can be verified by considering the table you constructed in part 1. In $\mathbb{Z}_4$, however, this is not always the case, if $a = b = 2 \in \mathbb{Z}_4$, then $2 \times 2 = 0$, but $2 \neq 0$ in $\mathbb{Z}_4$.

3. # 3 from the handout. The statement to prove should read: “If $n$ is prime and $a, b \in \mathbb{Z}_n$ with $ab = 0$, then either $a$ or $b$ is equal to zero. The proof is as follows.
Proof. Suppose $n$ is prime and that $ab \equiv 0 \mod n$. Then by definition, $n|(ab)$. Since $n$ is prime, this implies that $n|a$ or $n|b$. But this means that either $a$ or $b$ is congruent to 0 mod $n$.

4. # 3 from Chapter 1. No, $D_3$ is not abelian. Consider $R, F \in D_3$ where $R$ is the rotation of $120^\circ$ counterclockwise and $F$ is any reflection. Then $RF = FR^{-1} = FR^2$, and this cannot equal $FR$, as $FR^2 = FR$ if and only if $R^2 = R$, which it is most certainly not. So, since there are elements $a, b \in D_3$ so that $ab \neq ba$, we conclude that $D_3$ is nonabelian.

Remark. As a raccoon, I feel that it is my duty to pass along observations about the homework should I feel the need to. One observation I had was that the assignments are good, but there was something about this last problem that I’m not sure everyone quite grasped. Let me list them:

(a) Be sure to label everything and define everything. A lot of people agreed that $D_3$ is not abelian but mentioned elements $R$ and $F$ without properly defining them. How do I know that $RF \neq FR$ if I don’t know what the group elements are? Of course, many times it was clear from context, but for those who made an argument that, for instance, $RF = (\text{some group element } a)$ and $FR = (\text{some other group element } b)$, and conclude that $RF \neq FR$ since $a \neq b$, it was difficult to give 100% credit for this reasoning as it was impossible to check whether or not $RF$ actually equals $a$ and whether or not $FR$ actually equals $b$. As I said. This is minor, but consider it, and the small number of points missed, as a warning for future homework problems.

(b) Remember how one carries out the multiplication in $D_n$: the isometry $ab$ is equal to the composition of the isometry which is first $b$, then $a$. Along the same lines of correcting your work, it was apparent that some of you were applying the operation backwards. You all seemed to know the difference between $ab$ and $ba$ in that they are operations in opposite orders, but I think several of you computed $ab$ as the operation $ba$ and vice-versa. So be careful with that in the future. Other than that, ROCK ON!