Hi kids, here are some selected solutions to the first homework assignment. Enjoy!

1.2.3a Here are two such graphs that are not isomorphic:

![Graph A](image1)

![Graph B](image2)

are not isomorphic, since the former has a vertex of degree 5, and the latter does not.

1.2.5a These graphs are not isomorphic because the first graph does not have a cycle of length 5, while the second graph does. Another reason why is that the first graph is bipartite, and the second graph is not: one can check that there are only cycles of even length in the first graph (or alternatively, you can give the partitioning of the vertices to demonstrate that it is bipartite), and that there are cycles of odd length in the second graph (we have a theorem that says that as a result, such a graph could not be bipartite).

1.3.1 We use the fact that the sum of the degrees over all vertices is twice the number of edges. (a) We get $3v = 24$, so $v = 8$. (b) We get $4 + 4 + 4 + 3v = 42$, where there are $v + 3$ vertices (the three of degree 4, and the rest of degree 3). Thus, $v = 10$, and so the total number of vertices is 13. (c) We have $48 = dv$, where $d$ is the common degree of each vertex. There are a number of solutions to this, but they all share the same property that the number of vertices must be a divisor of 48.

1.3.9 We use the fact that the sum of the degrees over all vertices is twice the number of edges. This gives us $pv = 2e$, where $p$ is the odd number given, $v$ is the number of vertices, and $e$ is the number of edges. Notice that since $2e$ is even, either $p$ or $v$ is even (or both). But $p$ is given as odd, so $v$ must be even, and therefore $p \left( \frac{v}{2} \right) = e$, where $\frac{v}{2}$ is an integer. Thus, $e$ is a multiple of $p$.

1.4.8 We use the fact that the sum of the degrees over all vertices is twice the number of edges, and Euler’s formula $r = e - v + 2$. Summing over all degrees, we have $4n = 2e$, or that $e = 2n$. Then the number of regions $r = 10 = e - n + 2$ (remember, $n$ is given as the number of vertices), and $e = 2n$, so $10 = 2n - n + 2$, so that $n = 8$. 

1