Hi everyone. Please enjoy these solutions to your Quiz # 2.

(1) The cross product is
\[
\begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
0 & 1 & 3 \\
4 & 1 & 1 \\
\end{vmatrix} = \vec{i}(-2) - \vec{j}(-12) + \vec{k}(-4) = -2\vec{i} + 12\vec{j} - 4\vec{k}.
\]

(2) Consider approaching \((0, 0)\) along the path \((t, 0)\). We have \(f(t, 0) = \frac{t^3}{r^2 + t^2} = 1 \rightarrow 1\) as \(t \rightarrow 0\). Now approach the origin along the path \((0, t)\). We have \(f(0, t) = \frac{\cos \theta}{r^2 + t^2} = 0 \rightarrow 0\) as \(t \rightarrow 0\). Because these two outcomes are different, the limit must not exist.

(3) The vector tangent to the curve there is \(C'(1)\), and \(C'(t) = (2t, 2t^2, 3t^2)\), so \(C'(2) = (2, 4, 12)\). When \(t = 2\), the location in space is \(C(2) = (4, 3, 9)\), which is a point the plane goes through. So the equation of the plane through that point with normal vector \((2, 4, 12)\) is
\[
2(x - 4) + 4(y - 3) + 12(z - 9) = 0, \text{ or } 2x + 4y + 12z = 20.
\]

(4) The vector this line points in is perpendicular to both planes' normal vectors. These are \((1, 0, 1)\) and \((2, 0, -1)\). Such a vector perpendicular to both can be found by cross producting these:
\[
\langle 1, 0, 1 \rangle \times \langle 2, 0, -1 \rangle = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
1 & 0 & 1 \\
2 & 0 & -1 \\
\end{vmatrix} = \vec{i}(0) - \vec{j}(-3) + \vec{k}(0) = 3\vec{j}.
\]

So, this line points in direction \((0, 3, 0)\) and goes through \((3, 0, 1)\) as noted in the problem. So the parameterization is
\[
\ell(t) = (3, 0, 1) + t(0, 3, 0) = (3t, 3, 1).
\]

(5) For this function,
\[
\frac{\partial f}{\partial x} = 2xy + y \cos(xy).
\]