Hi everyone! I hope you enjoy these solutions! Rock on!

1. Let $u = x$, $dv = e^x dx$. Then $du = dx$, and $v = e^x$. And so we have

$$\int x e^x \, dx = x e^x - \int e^x \, dx = x e^x - e^x + C.$$  

2. We will use integration by parts twice. First, we let $u = x^2$, and $dv = \cos x \, dx$. Then $du = 2x \, dx$, and $v = \sin x$. So

$$\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx.$$

To integrate $\int 2x \sin x \, dx$, we let $u = 2x$, and $dv = \sin x \, dx$. Then $du = 2\, dx$, and $v = -\cos x$. So

$$\int 2x \sin x \, dx = -2x \cos x - \int -2 \cos x \, dx = -2x \cos x + \int 2 \cos x \, dx.$$

Plugging that into the above, we get

$$\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx = x^2 \sin x - (2x \cos x + \int 2 \cos x \, dx) = x^2 \sin x + 2x \cos x - 2 \sin x + C.$$  

3. We choose $u = e^x$, and $dv = \sin x \, dx$. Then $du = e^x \, dx$, and $v = -\cos x$. So

$$\int e^x \sin x \, dx = -e^x \cos x - \int -e^x \cos x \, dx = -e^x \cos x + \int e^x \cos x \, dx.$$

To integrate $\int e^x \cos x \, dx$, we choose $u = e^x$ and $dv = \cos x \, dx$. Then $du = e^x \, dx$, and $v = \sin x$. Then

$$\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx.$$

Plugging this into the top, we get

$$\int e^x \sin x \, dx = -e^x \cos x + (e^x \sin x - \int e^x \sin x \, dx)$$

now add $\int e^x \sin x \, dx$ to both sides:

$$2 \int e^x \sin x \, dx = -e^x \cos x + e^x \sin x,$$

so

$$\int e^x \sin x \, dx = \frac{1}{2} [-e^x \cos x + e^x \sin x].$$

ROCK ON.