Hi kids! Corey has spent the weekend trying to catch gophers on his property (among other things). I’m one of the gophers digging through Corey’s property that he’s trying to catch. He estimates that there could be as many as 12 of us and moles (left) on the property. I escaped tormenting him to momentarily write this review sheet. The following sections will be on the quiz. ROCK ON!

1. Section 2.3: Product and Quotient rules and higher-order derivatives. In this section you learn the product and quotient rules, which, combined with our other rules from the previous section, increase vastly the kinds of functions we can differentiate. For example, we can differentiate all sorts of trig functions, and rational functions (fractions of polynomials) as well.

2. Section 2.4: The Chain Rule. This section is very important. And I mean that! We’ve studied different types of differentiation formulae for sums, differences, products and quotients of functions. This is the last sort of technique you’ll need to know and it deals with composition of functions. It says:

\[
\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x).
\]
That means that whenever you encounter a function of the form \( f(g(x)) \), and are asked to take its derivative, then you must do the following: First, identify what function is \( f(x) \) and \( g(x) \). Then compute \( f'(g(x)) \) and \( g'(x) \). Finally, plug \( g(x) \) into \( f'(x) \) and put it all together as I have above. Corey’s explanation in class, the examples in class, the notes, the homework, and the book will all provide excellent resources.

3. Section 2.5: Implicit Differentiation. This section is really just an extension of the chain rule section, and I would keep that in mind as we study it. Really it provides a shortcut for computing derivatives and the only thing to keep in mind that problems from this section are really just applications of the chain rule, where the quantity \( y \) is the function named \( g(x) \) in the last section.

4. Section 2.6: Related Rates. This section is great. Actually, it can be annoying if you dislike story problems. But, finally, some story problems that are actually interesting! In this section one is given a description of some sort of situation. As time increases, there is usually some sort of movement described: something being filled with water, someone moving from 2nd to 3rd base, you watching your shadow move across the floor, etc. A working knowledge of the Pythagorean Theorem will be very helpful. There will likely be 2 or more quantities involved that you would implicitly assume are quantities that depend on time. The question might ask you about some rate of change, and it is your job to determine an equation that relates the rates involved, differentiate that relationship (usually implicitly) and solve for the desired rate.

5. Section 3.1: Extrema. In this section (and chapter, as a whole) we study functions through the eyes of derivatives. That is, we gather information about a function (and/or its graph) by studying the derivative of the function. We learned that critical points are of utmost importance. We also learned that if a function is continuous on a closed and bounded interval, then there exists a global (or absolute) maximum and minimum on the interval given—these must occur at critical points (by definition, the endpoints of the interval are considered critical), and hence the importance for their study. I would know how to find global extrema by considering these critical points.

6. Section 3.2: Rolle’s Theorem and the Mean Value Theorem. This section rules. It’s about two very important theorems in mathematics—these theorems are used all over the place and I wouldn’t be a very clever gopher if I didn’t point that out. I would know when you can apply these theorems and how to find the points they guarantee exist.

7. Section 3.3: Increasing and Decreasing functions and the 1st derivative test. This section discusses when functions are increasing and/or decreasing, again, through the use of the first derivative. The first derivative test is something that describes the
behavior of a function near a critical point, and thus, will help to identify local (or relative) extrema.

8. Section 3.4: Concavity and the Second Derivative Test. This section really completes our study of curve sketching, and our understanding of graphs of functions (and the behavior of functions, really). Just remember that the value of the second derivative can determine what exactly is going on at each critical point (unless $f''(x) = 0$ at said critical point). In any event, just remember that the second derivative measures the concavity of the function, similar to how the first derivative measures the slope of the tangent line.

9. Section 3.6: A summary of curve sketching. This section just puts everything we have learned from this chapter into use. There is nothing new to learn from this section, and Corey will likely ask you to use the techniques from this chapter to demonstrate an ability of curve sketching, besides just displaying individual skills from each of the sections from this chapter themselves. Corey would limit his curve sketching questions to polynomials.

10. General suggestions: Corey hopes that this quiz will be a straightforward experience for you, and suggests that you simply do as many practice problems as you feel comfortable with in order to grasp the material. Also keep in mind that the final exam will be cumulative, and will cover one or two more sections, but that there will also be a separate review sheet for the final as well. Good luck, and ROCK ON!