Solutions to Quiz # 1

The Oregon Duck

February 6, 2009

Hi college football fans!!! After Corey’s rant in class yesterday about the University of Florida and their football program, I thought I’d step in and write the solutions to the quiz you just took. Enjoy!

1. (a) The slope is \( (5 - 2)/(2 - 1) = 3 \). (b) The slope is \( (3.25 - 2)/(1.5 - 1) = 1.25/.5 = 2.5 \). (c) The slope is \( (2.21 - 2)/(1.1 - 1) = .21/.1 = 2.1 \). (d) Based on these answers, it seems that the slope of the tangent line at \( x = 2 \) is 2.

2. Let \( \epsilon > 0 \) be given. Then \( |(4x + 1) - 9| = |4x - 8| = 4|x - 2| < 4\delta = \epsilon \). Therefore, we set \( \delta = \epsilon/4 \).

3. According to the graph: (a) 1.75 (or anything relatively close to that), (b) Does not exist.

4. (a)
\[
\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = \lim_{x \to 4} \frac{(x + 4)(x - 4)}{x - 4} = \lim_{x \to 4} x + 4 = 8.
\]
(b) 
\[ \lim_{x \to 1} \sin(x^2 - 1 + \pi) = \sin \left( \lim_{x \to 1} x^2 - 1 + \pi \right) = \sin \pi = 0. \]

5. First, we compute
\[ f(x + \Delta x) = 2(x + \Delta x)^2 - 1 = 2(x^2 + 2x\Delta x + (\Delta x)^2) - 1 = 2x^2 + 4x\Delta x + 2(\Delta x)^2 - 1. \]

Then:
\[ f(x + \Delta x) - f(x) = 2x^2 + 4x\Delta x + 2(\Delta x)^2 - 1 - (2x^2 - 1) = 4x\Delta x + 2(\Delta x)^2. \]

Thirdly:
\[ \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{4x\Delta x + 2(\Delta x)^2}{\Delta x} = 4x + 2\Delta x. \]

Now finally, as \( \Delta x \to 0 \):
\[ \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} 4x + 2\Delta x = 4x. \]