1. Use remainder Thm. to find \( f(-3) \) if \( f(x) = x^3 + 2x^2 + 3x + 4 \). Check your answer.

\[
\begin{array}{cccc}
-3 & 1 & 2 & 3 & 4 \\
 & -3 & 3 & -18 & \\
1 & -1 & 6 & 1-14 & \text{Our remainder is -14 so} \\
\end{array}
\]

2. Find a polynomial \( f(x) \) with roots at 0, -2, 3 which satisfies \( f(1) = 30 \).

\[ f(x) = a(x-0)(x+2)(x-3) \]
\[ f(x) = a x(x+2)(x-3) \]

To find \( a \), use \( f(1) = 30 \)

\[ f(1) = a(1+2)(1-3) = 30 \]
\[ a(-1) = 30 \]
\[ a = -30 \]
\[ a = -5 \]

so \[ f(x) = -5x(x+2)(x-3) \]
\[ = -5x(x^2 - x - 6) \]
\[ = -5x(x^2 - x - 6) \]
\[ = -5x^3 + 5x^2 + 30x \]

3. Show that 2 is a zero of \( f(x) \) of mult. 2 and express \( f(x) \) as a product of linear factors.

\[
\begin{array}{c|cccc}
2 & 3 & -5 & -12 & +16 \\
 & 6 & 2 & -20 & -16 \\
3 & 1 & 8 & 0 & 0 & \text{so 2 is a zero of at least mult. 1} \]
\end{array}
\]

so now we know 2 is a zero of mult. 2.

\[ f(x) = (x-2)^2(3x^2 + 7x + 4) \]
\[ \text{or} \]
\[ f(x) = (x-2)^2(3x+1)(x+1) \]
4. Use Descartes' Rule of Signs to determine the number of possible positive, negative real, and nonreal complex solutions to
\[ f(x) = x^3 - 3x^2 + 2x - 6x + 3 \]

**Number of positive real solutions**
- Number of variations in sign of \( f(x) \) or is less than that by an even integer.
- So \( f(x) = x^3 - 3x^2 + 2x - 6x + 3 \)

Has 3 variations in sign, so there are 3, 1, or 0 positive real solutions.

**Number of negative real solutions**
- The number of variations in sign of \( f(-x) \)
- So \( f(-x) = (-x)^3 - 3(-x)^2 + 2(-x) - 6(-x) + 3 \)
  \[ = -x^3 - 3x^2 - 2x + 6x + 3 \]

Has 1 variation in sign, so there is 1 negative real solution.

To find nonreal complex solutions, we need the table.

<table>
<thead>
<tr>
<th># positive real solutions</th>
<th>4</th>
<th>2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td># negative real solutions</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td># nonreal complex solutions</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

5. Find a polynomial with real coefficients and leading coeff. 1 having the given zeros. Express \( f(x) \) as a product of linear and quadratic polynomials with real coefficients that are irreducible over the real numbers.

Zeros are \( 3, 0, 2+i \), degree 4.

Since \( 2+i \) is a zero, so is \( 2-i \).

So \( f(x) = 1(x - 3)(x - 0)(x - (2+i))(x - (2-i)) \)
\[ = (x - 3)x(x - 2 - i)(x - 2 + i) \]
\[ = (x - 3)x(x^2 - 2x + 2)(x^2 - 4x + 5) \]

So \( f(x) = x(x^3 - 4x^2 + 5) \)
6. Use Theorem on Rational Zeros of a polynomial to find all solutions to

\[ f(x) = x^4 + 3x^3 - 12x^2 - 6x + 20 = 0 \]

Possible solution are in form \( \frac{p}{q} \) with \( p \) being factors of 20 and \( q \) a factor of 1

\[ \frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20 \]

Now find solutions by trial and error.

\[ \begin{array}{cccc}
1 & 3 & -12 & -6 & +20 \\
1 & 1 & -9 & -6 & -14 \\
1 & 2 & -4 & -6 & 12 \\
1 & 3 & -2 & 8 & -10 \\
1 & 4 & -4 & 6 & 10 \\
\end{array} \]

So 2 is a solution.

\[ f(x) = (x-2)(x^3 + 5x^2 - 2x - 10) \]

Now find solutions to

\[ x^3 + 5x^2 - 2x - 10 \]

by trial and error.

\[ \begin{array}{cccc}
1 & 5 & -2 & -10 & +16 \\
1 & 2 & -4 & -10 & +16 \\
1 & 4 & -2 & 8 & -10 \\
1 & 5 & -2 & -10 & 12 \\
1 & 5 & -2 & -10 & 14 \\
\end{array} \]

So x = \pm 1 is a solution.

\[ f(x) = (x-2)(x-5)(x^2 - 2) \]

\[ = (x-2)(x+5)(x^2 - 2) \]

To get last 2 solutions

\[ \frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20 \]

\[ \begin{array}{cccc}
1 & 9 & 34 & 126 \\
1 & 5 & 9 & 126 \\
1 & 1 & 6 & 126 \\
1 & 0 & 1 & 126 \\
\end{array} \]

So 2, 5 is a solution.

\[ x = 2, x = 5, x = \pm \sqrt{2}, x = \pm \sqrt{3}, x = \pm \sqrt{5} \]
7. Use Theorem on Rational Zeros of a Polynomial
To find all solutions of
\[ f(x) = 2x^3 + 5x^2 - 9x^2 - 18x - 18 = 0 \]
\[ g(x) = x^2(2x^3 + 5x^2 - 9x - 18) = 0 \]

Use Theorem on
\[ 2x^3 + 5x^2 - 9x - 18 \]
possible values of core = factors of -18
\[ \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18 \]
possible values of \( a \) are factors of 2
\[ \pm 1, \pm 2 \]
possible \( c/d = \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18 \]
\[ \frac{1}{2}, \frac{1}{3}, \frac{1}{9}, \frac{1}{18}, \frac{-1}{2}, \frac{-1}{3}, \frac{-1}{9}, \frac{-1}{18}, \frac{1}{2}, \frac{1}{3}, \frac{1}{9}, \frac{1}{18}, \frac{-1}{2}, \frac{-1}{3}, \frac{-1}{9}, \frac{-1}{18} \]

By trial & error

\[ f(2) = 0 \] so 2 is a solution
\[ f(x) = x^2(x-2)(2x^2+9x+9) \]

now we can factor this
\[ a = 2, q = 9 \]
\[ x = \frac{-9 \pm \sqrt{81 - 4(2)(9)}}{2(2)} \]
\[ x = \frac{-9 \pm 3}{4} \]
\[ x = \frac{-3}{2}, x = \frac{-3}{2} \]

So \[ f(x) = x^2(x-2)(2x+3)(x+3) = 0 \]

\[ x = 0 \text{ (mult. 2)} \]
\[ x-2 = 0 \]
\[ 2x+3 = 0 \]
\[ x+3 = 0 \]
\[ x = 2, x = -\frac{3}{2}, x = -3 \]

The solutions to \( f(x) \) are \( \{ 0 \text{ (mult. 2)}, 2, -\frac{3}{2}, -3 \} \)
8. Find the domain of \( f(x) = \frac{x+2}{x^3-9x} \)

\( x^3 - 9x \neq 0 \)
\( x (x^2 - 9) \neq 0 \)
\( x (x-3) (x+3) \neq 0 \)

\( x \neq 0 \)
\( x - 3 \neq 0 \)
\( x + 3 \neq 0 \)
\( x \neq -3 \)

\[ D = (-\infty, -3) \cup (-3, 0) \cup (0, 3) \cup (3, \infty) \]

9. Find the domain of \( f(x) = \frac{\sqrt{3x+2}}{x^2-4} \)

Cond. 1
\( 3x + 2 \geq 0 \)
\( -2 \leq x \leq \frac{2}{3} \)
\( x + \frac{2}{3} \)

Cond. 2
\( x^2 - 4 \neq 0 \)
\( (x-2) (x+2) \neq 0 \)
\( x - 2 \neq 0 \)
\( x + 2 \neq 0 \)
\( x \neq 2 \)
\( x \neq -2 \)

\[ D = [-\frac{2}{3}, 2) \cup (2, \infty) \]

10. Find the domain of \( f(x) = \frac{x-1}{\sqrt{x+3}} \)

\( x + 3 \geq 0 \) (Note that \( x + 3 \) cannot equal zero since it is in the denominator)
\( -3 \leq x \leq 0 \)
\( x \geq -3 \)

\[ D = (-3, \infty) \]

Page 5
11. Sketch the graph of \( f(x) = x^2 + 2 \) making use of symmetry, shifting, stretching, compressing, or reflecting as needed. The first graph \( g(x) = x^2 \). The new graph \( f(x) = g(x) + 2 = x^2 + 2 \) can be obtained by shifting the graph of \( g(x) \) up 2 units. Add 2 to each y-coordinate.

12. Sketch \( f(x) = |x+2| \)

First graph \( g(x) = |x| \)
Then graph \( f(x) = g(x+2) = |x+2| \)
by shifting graph of \( g(x) \) 2 units to the left.

13. Sketch \( f(x) = \left( \frac{1}{2}x \right)^3 \)

First sketch \( g(x) = x^3 \)
Then graph \( f(x) = g \left( \frac{1}{2}x \right) = \left( \frac{1}{2}x \right)^3 \)
by multiplying every value of \( x \) by \( \frac{1}{2} \)
14. Sketch \( f(x) = -2x^2 \)

First graph \( g(x) = x^2 \)

Then graph
\[ f(x) = -2g(x) = -2x^2 \]
by multiplying each y-coord. by \(-2\)