Division algorithms

Many divisions of small numbers can be calculated by knowing the multiplication table: $72 \div 9 = 8$ because $8 \times 9 = 72$. Another method is needed for bigger numbers.

Suppose you had to decide how many bags it would take to pack 39 cookies in bags of 12. You could do it by packing the cookies, then counting the bags. Or you could do the same thing on paper by repeatedly subtracting 12 from 39. Keep subtracting until there are fewer than 12. The number of times 12 is subtracted is the quotient (the number of sets of 12 in 39). The number left after all the subtraction is the remainder.

$$\begin{array}{r} 89 \\ -\underline{12} \\ 27 \\ -\underline{12} \\ 15 \\ -\underline{12} \\ 3 \end{array}$$

This method can be justified in another way by remembering that multiplication is repeated addition. Since subtraction is the reverse of addition, division is repeated subtraction.

Problem: Compute by repeated subtraction: (a) $105 \div 28$ (b) $204 \div 11$

Repeated subtraction becomes too inefficient when one of the numbers is much bigger than the other. The second algorithm for division is repeated subtraction by multiples of 10.

Suppose you had 2550 cookies to pack in bags of 12. By thinking ahead, you realize that you need at least 100 bags, since there are more than 1200 cookies. Subtract 1200 cookies; there are 1350 left. Keep track of the number of bags (the number of sets of 12) on the right side. Now you see that you can fill another 100 bags; subtract another 100. Now, since there are 150 cookies left, you can't fill 100 more bags, but you can fill 10 bags, since you have more than 120 cookies. Subtract 120. Now you can't fill 10 more bags, but you can fill at least one. Keep subtracting 12 until you have less than 12 left. This is the remainder. Total the number of 12s you subtracted; this is the quotient.

$$\begin{array}{rrr} 12\overline{)2550} & & \\ -\underline{1200} & 100 & 12s \\ 1350 & & \\ -\underline{1200} & 100 & 12s \\ 150 & & \\ -\underline{120} & 10 & 12s \\ 30 & & \\ -\underline{12} & 1 & 12s \\ 18 & & \\ -\underline{12} & \underline{\ 1} & 12s \\ 6 & 212 & 12s \end{array}$$

Problem: Compute by repeated subtraction by multiples of 10: (a) $210 \div 9$ (b) $928 \div 32$

The final streamlined version of the repeated subtraction method is the usual division algorithm. In the previous methods, at each stage, you might have to subtact the same number as many as 9 times. In this method, you figure out in advance how many you will need to subtract, then subtract them all at once.

To divide 793 by 12, you figure out how many sets of 100 12s, or 10 12s, or 1 12 you can subtract. It turns out you can subtract 6 sets of 10 sets of 12 (that is, $6 \times 10 \times 12$ $6 \times (10 \times 12)$ $=6 \times 120$) or 6 120s. Write the 6 above the tens place of 783 to represent 60 12s ($6 \times 10 \times 12$=$(6 \times 10) \times 12 = 60 \times 12$). After subtracting 6 120s, which is 720, you have 63 left. You can subtract 5 sets of 12, which is 60. Write the 5 above the ones place. There is a remainder of 3.

$$\begin{array}{r} 65 \text{ r. } 3 \\ 12\overline{)783} \\ -\underline{720} \\ 63 \\ -\underline{60} \\ 3 \end{array}$$

The hard part of this method is estimating how many multiples you can subtract. Here is an example of how to estimate: $4963 \div 21$. What multiples of 21 should we start with? $21 \times 10 = 210$? $21 \times 100 = 2100$? $21 \times 1000 = 21000$? 21000 is too big, so start with 100s of 21s, which are 2100s. How many 2100s are in 4963? This is too hard to think about, so ask an easier question: how many 2000s in 5000? (Round off both numbers to one digit and 0s.) There are 2 2000s in 5000, so there are about 2 2100s in 4963. Try it: 2 2100s make 4200. Subtract from 4963 to get 763. This is not enough for another 2100, so 2 was the right guess.

$$\begin{array}{r} 2 \\ 21\overline{)4963} \\ -\underline{4200} \\ 763 \end{array}$$

$$\begin{array}{r} 23 \\ 21\overline{)4963} \\ -\underline{4200} \\ 763 \\ -\underline{630} \\ 133 \end{array}$$

In general, there are two other possibilities: 2 2100s could have been more than you can take from 4963; if so, 2 would be too high a guess. The other possibility is that the remainder is 2100 or more; then 2 would be too low a guess.

$$\begin{array}{r} 236 \text{ r. } 7 \\ 21\overline{)4963}\phantom{\text{ r. } 7} \\ -\underline{4200}\phantom{\text{ r. } 7} \\ 763\phantom{\text{ r. } 7} \\ -\underline{630}\phantom{\text{ r. } 7} \\ 133\phantom{\text{ r. } 7} \\ -\underline{126}\phantom{\text{ r. } 7} \\ 7\phantom{\text{ r. } 7} \end{array}$$

Move to sets of 10 21s. Instead of asking how many 210s in 763, ask how many 200s in 700. The answer is 3, so there are about 3 210s in 763. Write 3 in the tens place, since 3 210s is 30 21s. Check by multiplying and subtracting: $30 \times 21 = 630$, $763 - 630 = 133$. Finally, to find how many 21s are in 133, ask how many 2s in 13: about 6. Multiply and subtract: $6 \times 21 = 126$, $133 - 126 = 7$. So the number of 21s in 4963 is 236, with a remainder of 7.

To save writing, you can leave off the 0s if you are careful to write all the digits in the right places. If you leave off the 0s, you "bring down" the digits one at a time as you need them.