Homework #6, Math 301: Decimals and percents

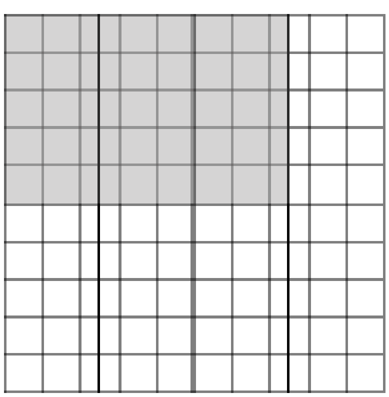
due Tues., May 12

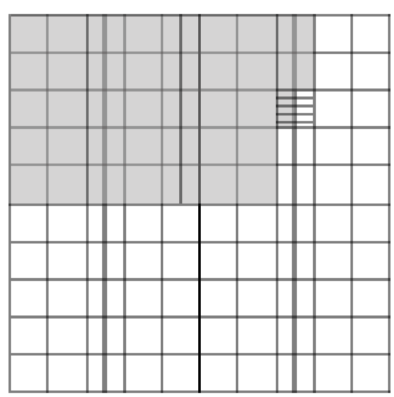
Decimals. Decimals are a way of expressing whole numbers, fractions, and even irrational numbers. Decimal numbers continue the place value system to the right of the decimal point. For instance, 34.82 means 3 tens + 4 ones + 8 tenths + 2 hundredths. So it's easy to write a decimal number as a fraction: just add the fractions for the place values: $30+4+\frac{8}{10}+\frac{2}{100}=34+\frac{80}{100}+\frac{2}{100}=34+\frac{82}{100}=34\frac{82}{100}$. You could also write it as an improper fraction:

$$30+4+\frac{8}{10}+\frac{2}{100}=\frac{3000}{100}+\frac{400}{100}+\frac{80}{100}+\frac{2}{100}=\frac{3482}{100}.$$

It's not so easy to write a fraction whose denominator is not a power of 10 as a decimal. For example, the fraction $\frac{3}{8}$ is measured in units of one eighth of a whole, and there are 3 of those units. Writing it as a decimal is asking to measure this same amount in tenths of a whole, hundredths of a whole, and so on.

The pictures show a whole big square divided into hundredths (the thin lines) and also divided into eighths (the thicker lines). The fraction 3/8 is shaded in both pictures. The pieces in the picture on the left have been rearranged to fit better into the grid of hundredths: 3/8 is equal to 37 hundredths plus half a hundredth, which is 5 thousandths. That is, $\frac{3}{8}=\frac{37}{100}+\frac{5}{1000}=\frac{375}{1000}=.375$





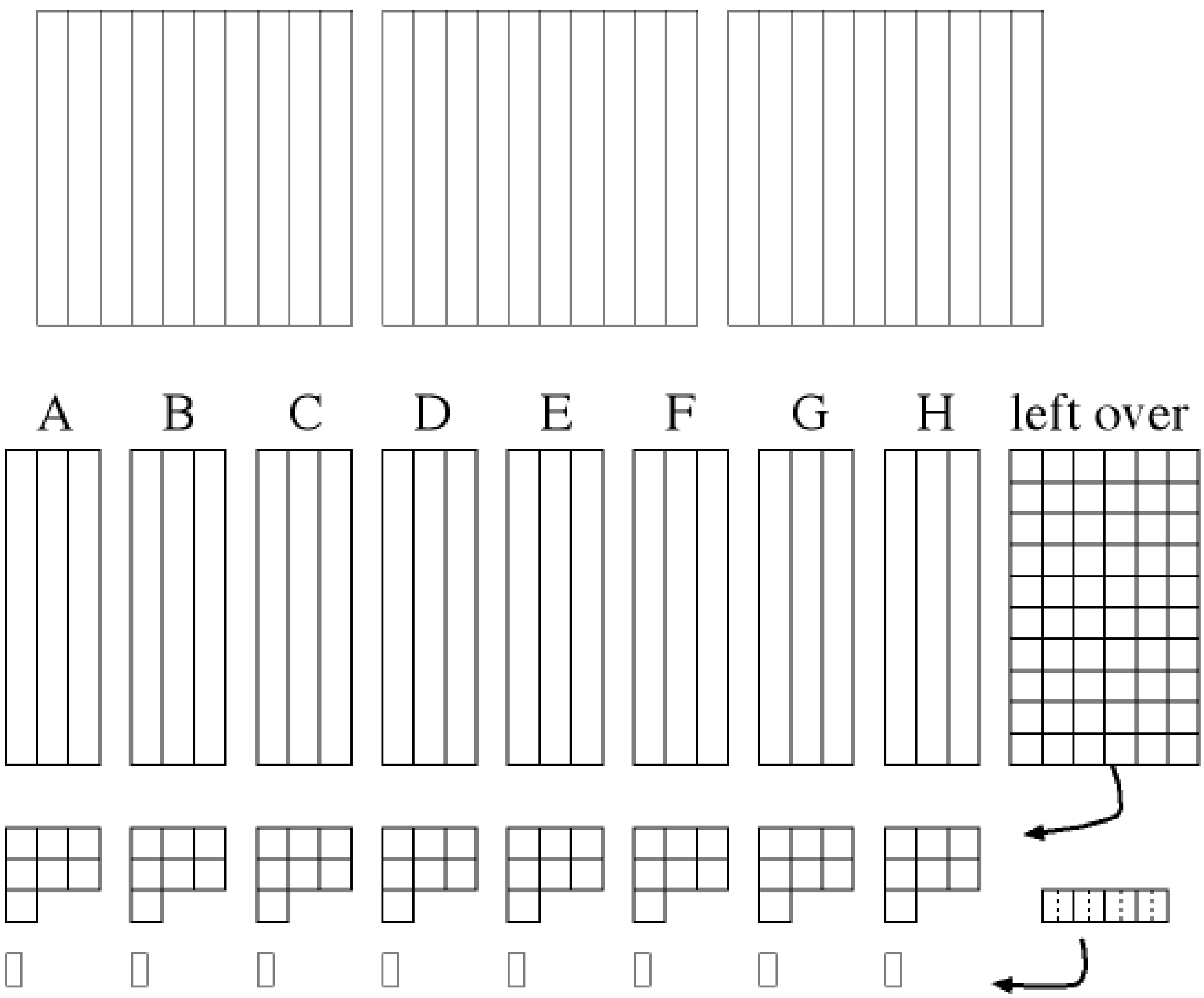
Here is another way to think of fractions as decimals, using a sharing/dealing idea of division. Recall that idea #5 of division is that a fraction is a division. Use the fraction 3/8 again, and think of it as $3\prod 8$. Now think of $3\prod 8$ as a sharing problem: share 3 big squares equally among 8 people. Accomplish this sharing by dealing out parts of the square, where the parts are tenths, hundredths, etc.

Step 1: Cut each of the 3 squares into tenths, and deal them out until you don't have enough to go around. (Each person will get 3 tenths, with 6 tenths left over.)

Step 2: Take each of the left over pieces, cut them into tenths, and deal them out. (Each person will get 7 hundredths, with 4 hundredths left over.)

Step 3: Repeat step 2 until either you have divided everything evenly, or until you have the accuracy you want (round off) or until you lose patience. (In this example, when you give each person 5 thousandths, it comes out even.)

So 3/8 is 3 tenths, 7 hundredths, and 5 thousandths, or .375. In the picture below, the thousandths are too small to see easily, so 5 thousandths is shown as half a hundredth.



A non-visual way to convert fractions to decimals is to do long division, continuing dividing after the ones place. Again we will use the example $3/8 = 3\prod 8$, but with a measurement story. You work in a factory packing bags of bird seed. The regular size bag holds 8 kilograms (kg), but there are also 1/10 size bags (8 tenths of a kg, or .8 kg), 1/100 size bags (.08 kg), etc. You are given 3 kg of seed to work on. The calculation below shows the repeated subtraction/ladder method, except instead of subtracting whole bags of seed, you subtract tenths, hundredths, and thousandths of a bag at a time. The process could be speeded up by filling several bags at once.

kg of seed	full-size bags		kg of seed	full-size bags		kg of seed	full-size bags
3			.44			.04	
-.8	.1		-.08	.01		-.008	.001
2.2			.36			.032	
-.8	.1		-.08	.01		-.008	.001
1.4			.28			.024	
-.8	.1		-.08	.01		-.008	.001
.6			.20			.016	
-.08	.01		-.08	.01		-.008	.001
.52			.12			.008	
-.08	.01		-.08	.01		-.008	.001
.44			.04			0	.375

The result says that 3/8 of a full size bag of seed is the same as .375 bag of seed.

Formats for quotients. In a division problem, the number you start with is called the dividend, the number you divide by is the divisor, the result is the quotient, and there may be a remainder. For example, in $17 \prod 3 = 5 \text{ r. } 2$, 17 is the dividend, 3 the divisor, 5 the quotient, and 2 the remainder.

When dividing whole numbers, there are several choices for the format of the result. The result in the example above has a *whole number quotient and remainder.* You could also express the answer as a (mixed) fraction: $17 \prod 3 = 5\frac{2}{3}$, or as a decimal: $17 \prod 3 = 5.666\ldots$ (The … means the 6s go on forever). Usually you round off a decimal to some appropriate number of decimal places. The form of answer should suit the problem. For instance, if you were sharing bowling balls, a fraction of a bowling ball wouldn't be a helpful answer. If you were cutting up apples to share, .6666… of an apple would be too hard to deal with.

Percent. Percent is a fraction whose denominator is 100. While fractions are sometimes given as pure numbers, such as 1/2, percents in problems are always percents **of** something. If a friend asked you "Can I have 50%?", you would ask, "50% of what?" Maybe she would like 50% of all your money, or 50% of the cookie you are eating, or 50% of the time you have available to listen to a problem of hers. In any case, 50% of a quantity means to divide the quantity into 100 equal parts, and take 50 of them. It would be less work to just divide it in 2, and take one.

Since a percent is a fraction, "of" means to multiply. So 50% of $142 means

$$\frac{50}{100} \quad \frac{\$142}{1} = \frac{1}{2} \quad \frac{\$142}{1} = \$71.$$

Often percents appear as percent more than, or percent less than something. For example, sales tax of 8% is 8% more than the purchase price: add 8% of the purchase price to the purchase price. Interest on a debt is similar: if the interest rate is 1.5% per month (which is 18% a year), after 1 month you will owe 1.5% of the money you originally owed, plus the money originally owed. Discounts are percent less than. At a sale of 30% off, you pay the regular price, minus 30% of the regular price.

In all these cases, a 2-step process can be done in one step.

Sales tax: The purchase price is 100% of the purchase price (duh!), and the tax is 8% of the purchase price. So altogether you pay 108% of the purchase price. In fractions, 100/100 of the price + 8/100 of the price is 108/100 of the price. You can think of this as measuring money where the unit is the purchase price. Here is a specific example: a T-shirt costs $10. Tax is 8% of $10, which is $.80. The total is $10.80, which is 108% of $10, which means $\frac{108}{100} \quad \$10$.

Discount: Suppose you are buying a non-taxable item whose regular price is $10. It is on sale for 30% off. You could compute 30% of $10 (which is $3) and subtract from $10 to get $7. Or do it in one step: 100% of the price – 30% of the price is 70% of the price. 70% of $10 is $7.

Rule: To get P% more than something, add P to 100 and multiply that percent by the something. To get P% less than something, subtract P from 100 and multiply that percent by the something.

Frequently, in real life, this process is repeated. If you do not pay your credit card balance for several months in a row, you owe not only the balance, but the interest, and the interest on the balance and interest, and so on. (In real life, you also owe fees, which makes it even more complicated.)

Example: Your credit card company does not charge any fees, but compensates by charging 2.5% interest each month. You don't pay your balance of $1000 for 4 months in a row. How much do you owe at the end? Each month, the company multiplies your previous balance by 100%+2.5%, or 102.5%, which is 1.025 as a decimal.

month	owed	how to calculate:
0	$1,000.00	
1	$1,025.00	$1000 x 1.025
2	$1,050.63	$1025 x 1.025
3	$1,076.89	$1050.63 x 1.025
4	$1,103.81	$1076.89 x 1.025

Homework

1) Write (and solve) three word problems about division with whole numbers: one each whose answer is appropriately given as
   a) a whole number quotient and remainder
   b) a mixed fraction
   c) a decimal
2) Suppose you have a calculator that doesn't have an integer division key, and you need to know the exact whole number quotient and remainder for 6589712 ∏357159. You use the regular division key, and the calculator says the answer is 18.450359. Find the exact whole number quotient and remainder without doing any calculations by hand, and describe your method. Explain how 18 and .450359 are related to the dividend, divisor, quotient, and remainder.
3) a) Use centimeter graph paper to make a 10 cm by 10 cm square. Share this square into three equal piles by first dealing out tenths, then hundredths, then thousandths. Put any amount left over into a separate pile at the end. Tape your work to a piece of paper so that it's clear what you did.
   b) Express the amount in each pile as a fraction, and as a decimal. Briefly explain.
4) Use long division (traditional or ladder) to express 2/7 as a decimal. Keep dividing until you see a pattern repeat.
5) A plant grows 10% every week. At week 0, it weighs 200 grams. Make a table showing how much it weighs each week from the 0^{th} week to the 8^{th} week.
6) At Filene's basement store, items that don't sell are marked down 10% at the end of the week. A weird pink and orange sweater originally cost $100, and hasn't sold in 8 weeks. Make a table showing its price each week from the 0^{th} week to the 8^{th} week.