

**MATH 270-01 (80278): Elementary Differential Equations
SYLLABUS, FALL 2009
TR 12-1:50 PM, JB-387**

John Sarli
JB-326
MW 5-6PM, TR 11AM-NOON
or by appointment
(909)537-5374
jsarli@csusb.edu

Text: Polking/Bogges/Arnold,
Differential Equations with Boundary Value Problems (2nd edition)
ISBN 0-13-186236-7

Prerequisites: MATH 252

This is a course in differential equations involving derivatives of functions of a single variable. Such equations are commonly called *ordinary differential equations* (ODE's). The emphasis is on techniques for finding solutions of single equations and examining the qualitative behavior of simple systems. Some theory will be presented but proofs of most theorems are generally to be found in a second course (such as MATH 470). In this text we will cover material selected from Chapters 1, 2, 3, 4, 7, 8 and 9.

Grading will be based on the following components, weighted as indicated:

- 1) First Assignment on Chapter 1 (10%)
- 2) Midterm on Chapter 2 (25%)
- 3) Second Assignment on Chapter 3 (20%)
- 4) Third Assignment on Chapters 7, 8 (20%)
- 5) Final Exam (25%)

Course grades will be determined, according to the above weighting, as follows:

<i>A</i>	≥ 91
<i>A-</i>	86 – 90
<i>B+</i>	81 – 85
<i>B</i>	76 – 80
<i>B-</i>	71 – 75
<i>C+</i>	66 – 70
<i>C</i>	61 – 65
<i>C-</i>	51 – 60
<i>D</i>	45 – 55
<i>F</i>	< 45

Below is a tentative schedule. Dates for the midterm exam and assignments are subject to change. A list of suggested exercises for each chapter in the text will be provided that should suffice for exam study. The three assignments are graded on presentation as well as mathematical content. Do not turn in

rough drafts of these and do not submit your only copy. (This may be a good opportunity to use mathematical software formatting; electronic submissions as .pdf files will be accepted.) If you are in need of an accommodation for a disability in order to participate in this class, please contact Services to Students with Disabilities at UH-183, (909)537-5238.

September 24: Overview; solution curves

September 29: Separable equations

September 30: Last day to add open classes via MyCoyote

October 1: Linear Equations; **First Assignment Due**

October 6: Related Systems

October 8: Exact Equations

October 13: Existence and Uniqueness

October 14: CENSUS (Last Day to Drop Without Record of Enrollment)

October 15: Dependence on Initial Conditions

October 20: **Midterm Exam**

October 22: Campus closed

October 26: Winter Advising begins

October 27: Autonomous Equations; Second Order Equations

October 29: Related Systems; **Second Assignment Due (Chapter 3)**

November 2: Grad Check Deadline for Spring (June) 2010

November 3: Constant Coefficients - Homogeneous Case

November 5: Inhomogeneous Case

November 10: Methods

November 11: Campus closed

November 12: Applications

November 17: Review of Matrix Algebra

November 19: Vector Spaces

November 24: Application to Linear Systems

November 26: Campus closed

November 30: Campus closed

December 1: Eigenvectors

December 3: Planar Systems; **Third Assignment Due**

December 8: **Final Exam**

Chapter 1 Assignment:

1.1 (pages 5-6): 2, 4, 6, 10

1.2 (pages 9-10): 14, 18

1.3 (pages 14-15): 6, 12, 20, 28

Suggested Exercises for Chapter 2

2.1 (pages 25-27): 1, 3, 9, 11, 13, 17, 19, 37

2.2 (pages 35-37): 1 – 17 odd, 23, 27

2.3 (pages 44-46): 1, 913

2.4 (pages 55-56): 1, 3, 5, 9, 13, 17, 21, 35, 39

- 2.5 (pages 61-63): 1, 5
- 2.6 (pages 75-77): 5, 9 – 21 odd, 25, 27, 35
- 2.7 (pages 86-87): 1, 2, 7, 23, 25, 27, 29, 31
- 2.8 (page 91): 17
- 2.9 (pages 100-102): 7, 9, 15, 17, 21 – 31 odd

Chapter 3 Assignment: *Choose the exercises from **one** of the following three sections. Submit your completed draft either electronically (pdf only), as a print-out, or in a bluebook.*

- 3.1 (pages 114-117): 6, 10, 12, 16, 20
- 3.3 (pages 126-127): 2, 4, 6, 10, 14
- 3.4 (pages 131-132): 4, 6, 12, 16, 22

Suggested Exercises for Chapter 4

- 4.1 (pages 145-146): 1 – 19, odd, 23, 25
- 4.2 (pages 149-150): 1 – 7 odd, 17
- 4.3 (page 156): 1, 5, 9, 15, 17, 19, 25, 27, 29
- 4.4 (pages 163-164): 7, 11, 13, 15, 21, 23
- 4.5 (pages 172-173): 1, 5, 11, 15, 19, 25, 37, 39
- 4.6 (page 177): 1, 5, 9, 13
- 4.7 (pages 185-187): 1, 9, 11, 13, 17, 21, 31

Third Assignment: *Only print-out, pdf, or bluebook accepted.*

- 7.1 (page 283): 54
- 7.2 (page 292): 38
- 7.3 (page 299): 8
- 7.4 (page 307): 20
- 7.5 (page 317): 32
- 7.6 (page 322): 34
- 7.7 (page 328): 4
- 8.1 (page 337): 16

Linear Mixture Equations

Let

$V(t)$ be liters of liquid in tank

$x(t)$ be grams of dissolved substance

Define

$c(t) = \frac{x(t)}{V(t)}$, concentration of solute

Suppose

constant intake of a l/s with constant concentration c_I

constant outlet of b l/s

Then

$V(t) = V_0 + (a - b)t$, $V(t) > 0$

$V'(t) = a - b$

ac_I g/s is rate at which solute is entering tank

$bc(t)$ g/s is rate at which solute is leaving tank.

$$x' = ac_I - b \frac{x}{V_0 + (a - b)t}$$

which has integrating factor

$$u = e^{\int_p \frac{b}{V_0 + (a-b)t} dt}$$

Case 1) $a = b$

Then $u = e^{\frac{b}{V_0}t}$ yields

$$ux = \int ac_I e^{\frac{b}{V_0}t} dt = \frac{ac_I V_0}{b} e^{\frac{b}{V_0}t} + C$$

$$x(t) = \frac{ac_I V_0}{b} + C e^{-\frac{b}{V_0}t}$$

and since $x_0 = c_0 V_0$ we have

$$C = V_0 \left(c_0 - \frac{ac_I}{b} \right).$$

Example.

$a = 4$ l/s = b

$c_I = 5$ g/l

$V_0 = 500$ l

$c_0 = 3$ g/l

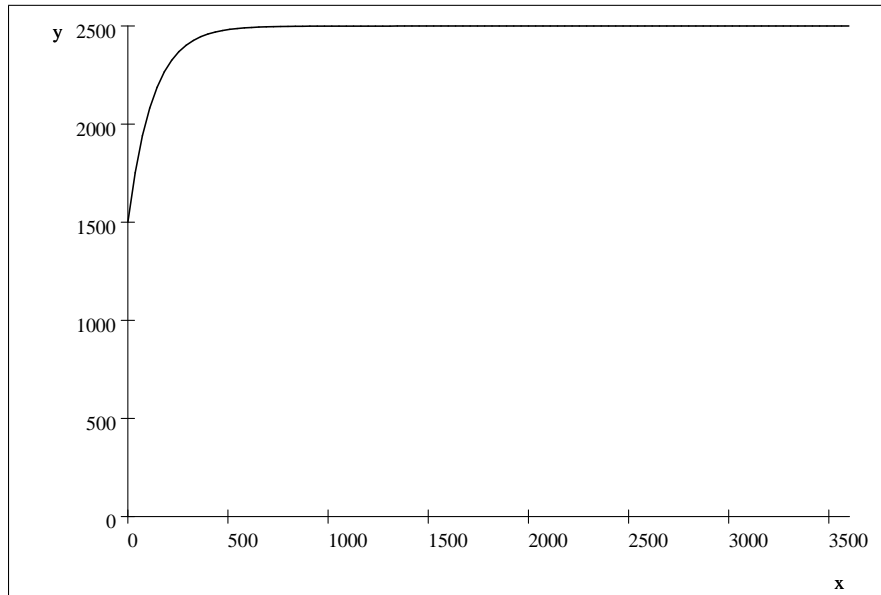
Then

$$C = 500(3 - 5) = -1000$$

$$x(t) = 2500 - 1000e^{-\frac{1}{125}t}$$

$$x(t) = 2500 - 1000e^{-\frac{1}{125}t}$$

$$x'(t) = 8e^{-\frac{1}{125}t}$$



Note:

$$x_0 = 1500 \text{ g}$$

$$V(t) = 500$$

$$c(t) = \frac{1}{500}x(t)$$

$$c(3600) = 5.0000$$

$$c(t) = 4, \text{ Solution is: } t = 86.643$$

Case 2) $a \neq b$

$$x'(t) = 30 - \frac{4x(t)}{500 + 2t}$$

$$x' + \frac{2}{t+250}x = 30$$

$$u(t) = e^{\int \frac{2}{t+250} dt} = (t + 250)^2$$

$$\int u(t)dt = \frac{1}{3} (t + 250)^3$$

$$ux = 10 (t + 250)^3 + C$$

$$x(t) = 10 (t + 250) + C (t + 250)^{-3}$$

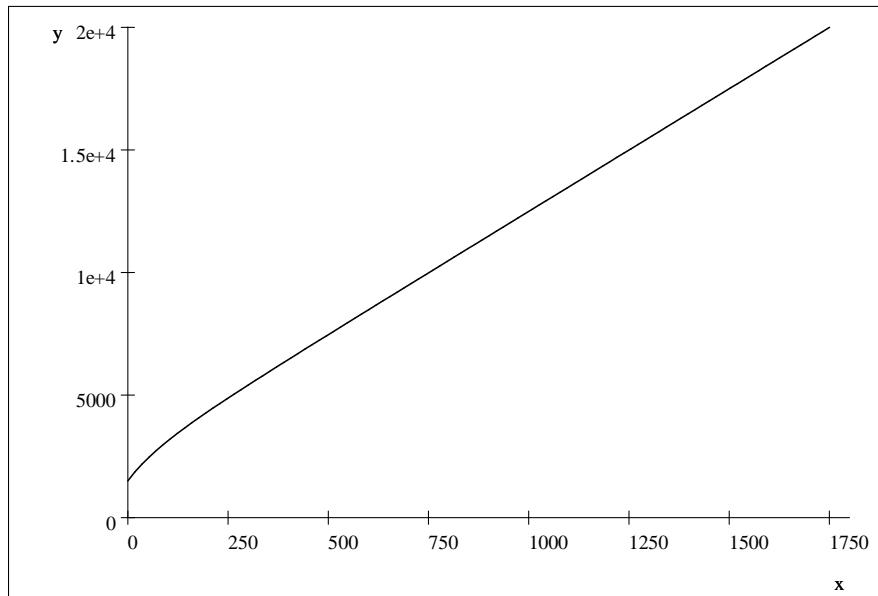
$$x(0) =$$

$$\frac{1}{15\,625\,000}C + 2500 = 1500, \text{ Solution is:}$$

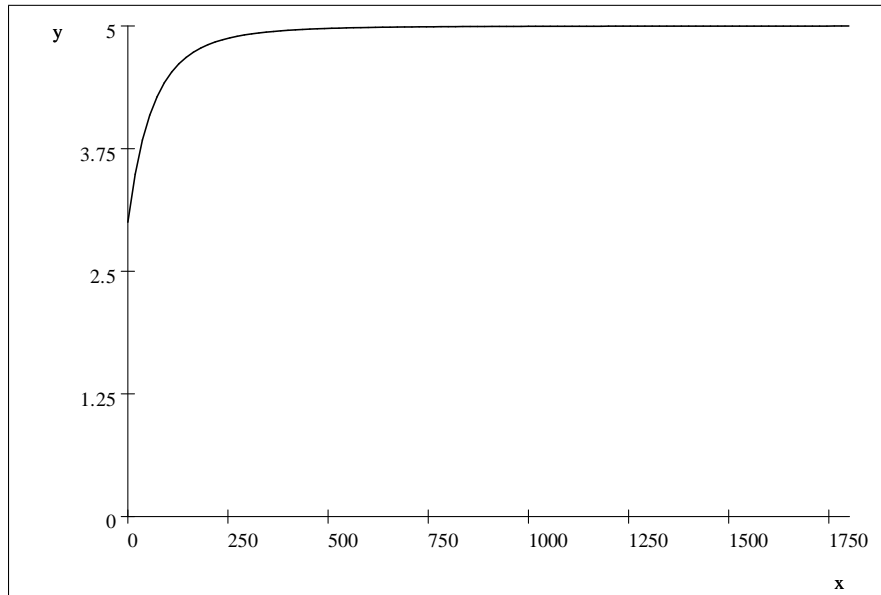
$$C = -15\,625\,000\,000$$

$$x(t) = 10 (t + 250) + C (t + 250)^{-3}$$

$$x(240) = \frac{560\,855\,100}{117\,649} = 4767.2$$



$$c(t) = \frac{x(t)}{500+2t}$$



$$c(250) = \frac{39}{8} = 4.875$$

Second tank:

$$y' = \frac{2}{250+t}x - \frac{1}{25}y = \left(\frac{2}{250+t}\right)x(t) - \frac{1}{25}y$$

$$y' + \frac{1}{25}y = \left(\frac{2}{250+t}\right)x(t) =$$

$$f(t) = 20(t+250)^{-4} (62\,500\,000t + 375\,000t^2 + 1000t^3 + t^4 + 2343\,750\,000)$$

$$u(t) = e^{\frac{1}{25}t}$$

$$e^{\frac{1}{25}t}y = \int u(t)f(t)dt$$

$$y(t) =$$

$$20e^{-\frac{1}{25}t} \int_p \frac{(t+250)^4 - 1562\,500\,000}{(t+250)^4} e^{\frac{1}{25}t} dt + Ce^{-\frac{1}{25}t}$$

$$= 500 + Ce^{-\frac{1}{25}t} - 20e^{-\frac{1}{25}t} \int_p \frac{1562\,500\,000}{(t+250)^4} e^{\frac{1}{25}t} dt$$

$$= 500 + Ce^{-\frac{1}{25}t} - 2^7 5^{12} e^{-\frac{1}{25}t} \int_p \frac{1}{(t+250)^4} e^{\frac{1}{25}t} dt$$

$$= 500 + e^{-\tau} \left(C - 2 \cdot 10^6 \int_p \frac{1}{\tau^4} e^{\tau} d\tau \right)$$

$$\tau = 10 + \frac{1}{25}t$$

$$\frac{1}{\tau^4} e^\tau = \int \left(\frac{1}{\tau^4} + \frac{1}{\tau^3} + \frac{1}{2\tau^2} + \frac{1}{6\tau} + \frac{1}{24} + \frac{1}{120}\tau + \frac{1}{720}\tau^2 \right) d\tau =$$

$$\tau = 10 + \frac{1}{25}t$$

$$t = 0$$

$$\tau = 10$$

$$y(\tau) = 500 + e^{-\tau} \left(C - \frac{25 \cdot 000}{27} \tau^{-3} (90\tau^4 - 1080\tau^2 - 1080\tau + 9\tau^5 + \tau^6 + 360\tau^3 \ln \tau - 720) \right)$$

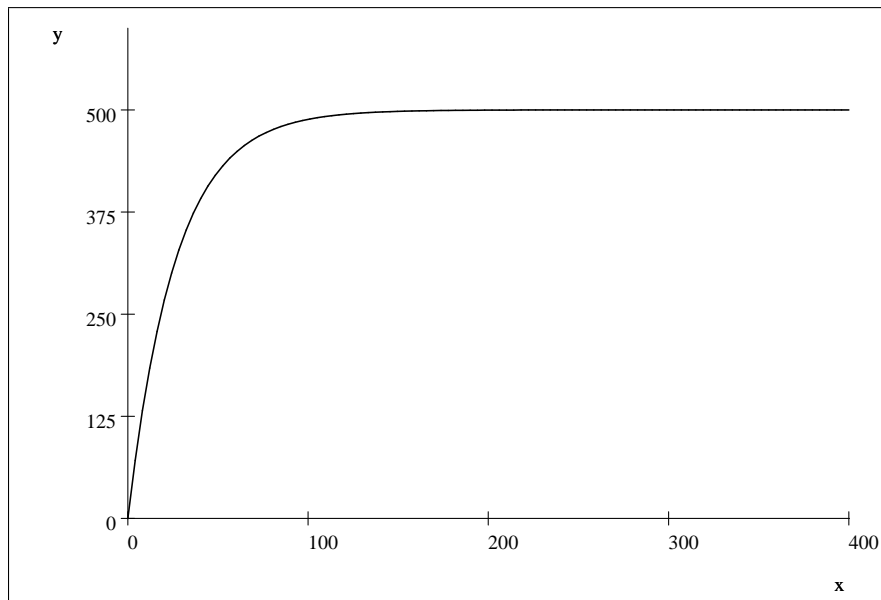
$$C = -7.7637 \times 10^6$$

$$y(13) = e^{-13} \left(-\frac{1000000}{3} \ln 13 - 1.2206 \times 10^7 \right) + 500 = 470.48$$

$$y(11) = 305.91$$

$$y(10.7321) = 249.99$$

$$10 + \frac{1}{25}t = 10.7321, \text{ Solution is: } \{[t = 18.303]\}$$



Variation of Parameters

To find a solution for

$$y''(t) + 4y'(t) + 3y(t) = 2te^t \cos 2t$$

we try a solution of the form

$$y_f(t) = e^t ((at + b) \cos 2t + (ct + d) \sin 2t)$$

Then

$$y_f''(t) + 4y_f'(t) + 3y_f(t) = 2e^t ((3c - 6b - 2a + 2d - 6at + 2ct) \sin 2t + (3a + 2b + 2c + 6d + 2at + 6ct) \cos 2t)$$

Since $f(t) = 2te^t \cos 2t$ we must have

$$(3c - 6b - 2a + 2d) + t(2c - 6a) = 0$$

and

$$(6a + 4b + 4c + 12d) + t(4a + 12c - 1) = t$$

and so

$$c - 3a \equiv 0$$

$$4a + 12c - 1 \equiv 1$$

$$3c - 6b - 2a + 2d \equiv 0$$

$$6a + 4b + 4c + 12d \equiv 0$$

This system is easily solved by back substitution since

$$c = 3a$$

then

$$a = \frac{1}{20}$$

$$c = \frac{3}{20}$$

from which the next two equations become

$$120b - 40d = 7$$

$$40b + 120d = -9$$

and so

$$b = \frac{3}{100}$$

$$d = -\frac{17}{200}$$

$$\text{Finally, } y_f(t) = e^t \left(\left(\frac{1}{20}t + \frac{3}{100} \right) \cos 2t + \left(\frac{3}{20}t - \frac{17}{200} \right) \sin 2t \right).$$