

6.5 Counterexamples

If " $\forall x \in S, P(x)$ " is not true, then its negation $\sim (\forall x \in S, P(x))$ is true. i.e., " $\exists x \in S, \sim P(x)$ " is true.

(Def) Any element $x \in S$ such that $\sim P(x)$ is true (or such that $P(x)$ is false) is called a **counterexample** of the statement " $\forall x \in S, P(x)$ ". i.e., A counterexample is an example illustrating that a "for all" statement is false.

Ex6.8 Consider the statement: If $x \in \mathbb{Z}$, then $\frac{x^3+x}{x^4-1} = \frac{x}{x^2-1}$.

Or equivalently, $\forall x \in \mathbb{Z}, \frac{x^3+x}{x^4-1} = \frac{x}{x^2-1}$.

This statement is _____ and _____ is a counterexample since

Ex6.32 Prove the following statement is false:

Let $a, b, c \in \mathbb{Z}$. If ab, ac and bc are even $\implies a, b, c$ are even.

(Pf):