

6.3 Existence Proofs

(Def) In an existence theorem the existence of an object possessing some specified property or properties is asserted.

A proof of an existence theorem is an **existence proof**. Such a proof may consist of displaying or constructing an example of such an object or perhaps, with the aid of known results, verifying that such objects must exist without ever producing a single example of the desired type.

R6.2 \Rightarrow Ex6.4 Prove: $\exists a, b, c \in \mathbb{R}$ such that $\frac{a+b}{a+c} = \frac{b}{c}$.
(Pf):

R6.3 Prove: $\exists a, b \in \mathbb{R}$ such that $(a + b)^2 = a^2 + b^2$.
(Pf):

(6.3 cont.)

R6.4 Prove: \exists irrational numbers a and b such that a^b is rational.
(*Pf*):

(6.3 cont.)

The **Intermediate Value Theorem** of Calculus

If f is continuous on $[a, b]$ and k is a number between $f(a)$ and $f(b)$
 $\implies \exists c \in (a, b)$ such that $f(c) = k$.

R6.5 \implies Ex Prove that the equation $3x^4 - 2x^3 + 5x - 5 = 0$ has a real number solution between $x = 0$ and $x = 1$.
(Pf):