## 4.3 Proofs involving Real Numbers

Some properties of $\mathbb{R}$ :
1. $\forall a \in \mathbb{R}$ and $n$: positive even integer$\Rightarrow a^n \geq 0$ .
2. If $a < 0$ and $n$: positive odd integer$\Rightarrow a^n < 0$ .
3. $\forall a.b \in \mathbb{R}, ab > 0 \Longleftrightarrow (a > 0$ and $b > 0)$ or $(a < 0$ and $b < 0)$.
4. Let $a, b, c \in \mathbb{R}$
(i) if $a \geq b$ and $c \geq 0 \Longrightarrow ac \geq bc$.
(ii) if $a \geq b$ and $c > 0 \Longrightarrow a/c \geq b/c$.
(iii) if $a > b$ and $c > 0 \Longrightarrow ac > bc$ and $a/c > b/c$.
(iv) if $a > b$ and $c < 0 \Longrightarrow ac < bc$ and $a/c < b/c$.

Thm 4.12 If $x, y \in \mathbb{R}$ s.t. $xy = 0 \Longrightarrow x = 0$ or $y = 0$.
$\langle Pf \rangle :$

R4.13$\Rightarrow$ *Ex*4.13 Let $x, y \in \mathbb{R}$.
Prove that if $x^2 - 4x = y^2 - 4y$ and $x \neq y$
$\Longrightarrow x + y = 4$.
$\langle Pf \rangle :$

(4.3 cont.)

Read R4.14

$\langle Def \rangle$ $\forall x \in \mathbb{R}$, its **absolute value** $|x|$ is defined as

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Thm 4.15 (**Triangle Inequality**)
$\forall x, y \in \mathbb{R} \Longrightarrow |x+y| \leq |x| + |y|$.
$\langle Pf \rangle$ : read for homework.

*Ex*4.19 $\forall x, y \in \mathbb{R} \Longrightarrow |x+y| \geq |x| - |y|$.
$\langle Pf \rangle$ :