4.1 Proofs involving Divisibility of Integers

⟨*Def*⟩ $a(\neq 0),\ b \in \mathbb{Z}$, we say that a **divides** b if $\exists c \in \mathbb{Z}$ such that $b = ac$. In this case, we write $a|b$.

e.g., n is an even integer$\Leftrightarrow$

p.62 R3.14 can therefore be restated for $a, b \in \mathbb{Z}$ as:

$$2|ab \Longleftrightarrow 2|a \quad \text{or} \quad 2|b.$$

⟨*Def*⟩ If $a|b$, then we also say that b is a **multiple** of a and that a is a **divisor** of b.

If a does not divide b, then we write $a \nmid b$.

e.g.,

R4.1 Let $a(\neq 0),\ b(\neq 0),\ c \in \mathbb{Z}$. If $a|b$ and $b|c \Longrightarrow a|c$.
⟨*Pf*⟩ :

(4.1 cont.)

R4.2 Let $a(\neq 0),\ b(\neq 0),\ c,\ d \in \mathbb{Z}$. If $a|c$ and $b|d \Longrightarrow ab|cd$.
$\langle Pf \rangle$:

R4.3 Let $a(\neq 0),\ b,\ c,\ x,\ y \in \mathbb{Z}$. If $a|b$ and $a|c \Longrightarrow a|(bx + cy)$.
$\langle Pf \rangle$:

R4.4 Let $x \in \mathbb{Z}$. If $2|(x^2 - 1) \Longrightarrow 4|(x^2 - 1)$.
$\langle Pf \rangle$:

(4.1 cont.)

R4.5 Let $x, y \in \mathbb{Z}$. If $3 \nmid xy \Longrightarrow 3 \nmid x$ and $3 \nmid y$.
$\langle Pf \rangle:$

R4.6⇒ *Ex*4.6 Let $a \in \mathbb{Z}$. Then $3|2a \Longrightarrow 3|a$.
$\langle Pf \rangle:$

(4.1 cont.)

<u>R4.7</u> Let $x, y \in \mathbb{Z}$. Then $4|(x^2 - y^2) \iff x$ and y are of the same parity.
$\langle Pf \rangle :$