

### 3.3 Proof by Contrapositive

*(Def)* The **contrapositive** of the implication  $P \Rightarrow Q$  is the implication  $\sim Q \Rightarrow \sim P$ . (they are logically equivalent).

$P$	$Q$	$\sim P$	$\sim Q$	$P \Rightarrow Q$	$\sim Q \Rightarrow \sim P$

So if it can be shown that  $\sim Q \Rightarrow \sim P$  is true, then it follows that  $P \Rightarrow Q$  is true as well.

*(Def)* A **proof by contrapositive** of  $P \Rightarrow Q$  is a direct proof of its contrapositive  $\sim Q \Rightarrow \sim P$ .

p.57 R3.7  $\Rightarrow$  Ex3.4 Let  $x \in \mathbb{Z}$ . Prove that if  $7x + 5$  is odd  $\Rightarrow x$  is even.  
*(Pf)*:

p.58 R3.8  $\Rightarrow$  Ex3.6 Let  $x \in \mathbb{Z}$ . Prove that  $5x - 11$  is even  $\Leftrightarrow x$  is odd.  
*(Pf)*:

(3.3 cont.)

**Thm 3.9** Let  $x \in \mathbb{Z}$ . Then  $x^2$  is even  $\iff x$  is even.  
(Pf):

Ex Let  $x \in \mathbb{Z}$ . Then  $x^2$  is odd  $\iff x$  is odd.  
Prove it for homework and compare it to the proof of Thm3.9.  
This simply restates Thm3.9 in terms of contrapositives.

p.59 R3.11 Let  $x \in \mathbb{Z}$ . Prove that if  $5x - 7$  is odd  $\implies 9x + 2$  is even.  
(Pf):