3.2 Direct Proofs

How to give a "**direct proof**" of $P \Rightarrow Q$?

We assume that $P(x)$ is true for some arbitrary element $x \in S$ and show that $Q(x)$ is true for this element x.

i.e., assuming that the hypothesis is true and then showing that the conclusion is true as well.

Useful properties:

1. $\forall n \in \mathbb{Z},\ -n \in \mathbb{Z}$.
2. $\forall m, n \in \mathbb{Z},\ m \pm n \in \mathbb{Z}$.
3. $\forall m, n \in \mathbb{Z},\ mn \in \mathbb{Z}$.

$\langle Def \rangle$

·The set of all **even** integers= $E = \{2k : k \in \mathbb{Z}\} = \{..., -4, -2, 0, 2, 4, ...\}$

·The set of all **odd** integers= $T = \{2k+1 : k \in \mathbb{Z}\} = \{2k-1 : k \in \mathbb{Z}\} = \{..., -3, -1, 1, 3, 5, ...\}$

$\langle Note \rangle$ $E \cup T = \mathbb{Z}$

i.e., every integer is either even or odd.

p.55 R3.3⇒ *Ex* If n is an odd integer$\Longrightarrow$ $5n-1$ is an even integer.

$\langle Pf \rangle:$

p.55 R3.4⇒ *Ex*3.2 If x is an even integer$\Longrightarrow$ $5x-3$ is an odd integer.

$\langle Pf \rangle:$

(3.2 cont.)

p.56 R3.5 If n is odd$\Longrightarrow 4n^3 + 2n - 1$ is odd.
$\langle Pf \rangle:$

p.56 R3.6$\Rightarrow$ Ex If n is an even integer$\Longrightarrow n^2$ is even.
$\langle Pf \rangle:$