3.1 Trivial and Vacuous Proofs

Let $P(x)$ and $Q(x)$ be open sentences, where x represents an element of some set S under discussion.

$P \Rightarrow Q$ is consider to be $\begin{cases} \textbf{true} \text{ provided } \forall x \in S, P(x) \Rightarrow Q(x) \text{ is true.} \\ \textbf{false} \text{ provided } \exists x \in S \text{ s.t. } P(x) \Rightarrow Q(x) \text{ is false.} \end{cases}$

If "$Q(x)$ is true $\forall x \in S$" or "$P(x)$ is false $\forall x \in S$", then determining the truth or falseness of $P \Rightarrow Q$ is simpler. (think about the truth table for "$\Rightarrow$")

If "$Q(x)$ is true $\forall x \in S$", then $P \Rightarrow Q$ is __________.
This constitutes a proof of $P \Rightarrow Q$ and is called a trivial proof.

e.g., "if $n^3 > 0$, then 3 is odd" is _________. And a trivial proof consists only of observing that 3 is an odd integer.

Vacuous Proof: prove $P \Rightarrow Q$ is true by showing that $P(x)$ is false $\forall x \in S$.

e.g., Let $x \in \mathbb{R}$. If $x^2 - 2x + 2 \leq 0$, then $x^3 \geq 8$.
$\langle Pf \rangle$:

$\langle Note \rangle$ In the end of a proof, mathematician usually use the following symbols to indicate the proof is complete: