2.9 Some Fundamental Properties of Logical Equivalence

· P and $\sim (\sim P)$ are logically equivalent.

· $P \wedge Q$ and $Q \wedge P$ are logically equivalent.
"$P \wedge Q$ is equivalent to $Q \wedge P$" is true
$(P \wedge Q) \Leftrightarrow (Q \wedge P)$ is a tautology

Thm 2.3 For statements P, Q, and R,

(1) Commutative Laws
(a) $P \vee Q$ is equivalent to $Q \vee P$
(b) $P \wedge Q$ is equivalent to $Q \wedge P$

(2) Associative Laws
(a) $P \vee (Q \vee R)$ is equivalent to $(P \vee Q) \vee R$
(b) $P \wedge (Q \wedge R)$ is equivalent to $(P \wedge Q) \wedge R$

(3) Distributive Laws
(a) $P \vee (Q \wedge R)$ is equivalent to $(P \vee Q) \wedge (P \vee R)$
(b) $P \wedge (Q \vee R)$ is equivalent to $(P \wedge Q) \vee (P \wedge R)$

(4) DeMorgan's Laws
(a) $\sim (P \vee Q)$ is equivalent to $(\sim P) \wedge (\sim Q)$
(b) $\sim (P \wedge Q)$ is equivalent to $(\sim P) \vee (\sim Q)$

Homework: Verify each part of Thm 2.3 by means of a truth table.

Ex Prove that $\sim (P \Rightarrow Q)$ and $P \wedge (\sim Q)$ are logically equivalent.
$\langle Note \rangle$ the negation of $P \Rightarrow Q$ is not an implication.