2.8 Logical Equivalence

⟨*Def*⟩ Whenever two (compound) statements R and S have the same truth value for all combinations of truth values of their component statements, then we say that R and S are **logically equivalent**.

e.g.,
1. $P \wedge Q$ and $Q \wedge P$ are logically equivalent.

2. (Thm 2.2) $P \Rightarrow Q$ and $(\sim P) \vee Q$ are logically equivalent.

P	Q			

Consider:

(I) Suppose R and S are logically equivalent,
then $R \Leftrightarrow S$ is true (since they have the same truth values)
then $R \Leftrightarrow S$ is a tautology.

(II) If $R \Leftrightarrow S$ is a tautology,
then R and S are logically equivalent.

Hence, two compound statements R and S are logically equivalent if and only if the statement "R is equivalent to S" is true.

Application Let R represent a mathematical statement that we would like to show is true, and suppose that R and S are logically equivalent for some statement S. If we can show that S is true, then R is true as well.

e.g., we want to prove the truth of $P \Rightarrow Q$. If we can prove the truth of $(\sim P) \vee Q$, then the logically equivalence of $P \Rightarrow Q$ and $(\sim P) \vee Q$ tells us that $P \Rightarrow Q$ is true as well.