2.11 Quantified Statements and Their Negations

If x is a real number, then $x^2 \geq 0$
$\approx$The square of every real number is nonnegative
$\approx$For every real number x, we have $x^2 \geq 0$.

Universal Quantifier: $\forall$
"every", "for every", "for each", "for all".

Notice that this is an upside down letter A, and that the letter A is the first letter in the word "all".
e.g., we can rewrite above:

If we define $P(x) : x^2 \geq 0$, then we can rewrite above:

Consider $Q(x) : x^2 \leq 0$, the statement $\forall x \in \mathbb{R}, Q(x)$ is _________.
i.e., there must exist some $x \in \mathbb{R}$ s.t. $x^2 > 0$.

Existential Quantifier: $\exists$
"there exists", "there is", "for some", "for at least one".

Notice that this is a backwards letter E, and that the letter E is the first letter in the word "exists".

we can rewrite "there must exist some $x \in \mathbb{R}$ s.t. $x^2 > 0$." by

(2.11 cont.)

If we are considering some universal set S and $P(x)$ is an open statement concerning some element $x \in S$, then

$$\boxed{\sim(\forall x \in S, P(x)) \text{ is logically equivalent to } \exists x \in S, \sim P(x).}$$

i.e., the opposite of "true for all" is "false for at least one".

e.g., Let A be a set. The negation of the statement "for every set $B \in P(A)$, $A - B \neq \varnothing$" is

$$\boxed{\sim(\exists x \in S, Q(x)) \text{ is logically equivalent to } \forall x \in S, \sim Q(x).}$$

i.e., the opposite of "true for at least one" is "false for all".

e.g., The negation of the statement "there exists a real number x s.t. $x^2 = 3$" is