1.6 Partitions of Sets

⟨Re*call*⟩ A and B are disjoint$\Longleftrightarrow$

⟨*Def*⟩ $S = \{\text{some subsets of a set } A\}$ is called **pairwise disjoint** if every two distinct subsets that belong to S are disjoint.

Ex $A = \{1, 2, 3, ..., 7\}$, $B = \{1, 6\}$, $C = \{2, 5\}$, $D = \{4, 7\}$
Let $S = \{B, C, D\}$, is S a pairwise disjoint collection of subsets of A? Why?

Ex $A' = \{1, 2, 3\}$, $B' = \{1, 2\}$, $C' = \{1, 3\}$, $D' = \{2, 3\}$
Let $S' = \{B', C', D'\}$, is S' a pairwise disjoint collection of subsets of A'? Why?

⟨*Def*⟩ A **partition** of A is a collection S of nonempty subsets of A s.t. every element of A belongs exactly one subset in S.

i.e., A partition of A can be def. as a collection S of subsets of A s.t.

$$\begin{cases} (1) \text{ if } X \in S \Longrightarrow X \neq \varnothing \\ (2) \text{ if } X, Y \in S \Longrightarrow X = Y \text{ or } X \cap Y = \varnothing \\ (3)\ A = \bigcup_{X \in S} X \end{cases}$$

Tip: think about "ways to divide a set into groups".

(1.6 cont.)

<u>*Ex*</u> $A = \{1, 2, 3, 4, 5\}$

Are the following partitions of A? Why or why not?

$S_1 = \{\{1, 2\}, \{3\}, \{4, 5\}\}$

$S_2 = \{\{1, 2\}, \{4, 5\}\}$

$S_3 = \{\varnothing, \{1, 2, 3\}, \{4, 5\}\}$

$S_4 = \{\{1, 2\}, \{2, 4\}, \{3, 5\}\}$

$S_5 = \{A\}$

$S_6 = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$

$S_7 = \{\{1, 2, 3\}, \{4, 5, 6\}\}$