Homework #6

Math 553 DUE: December 1, 2009

Exercises 3.2 (page 107): Problems:

1. Let $A = \{\frac{1}{n} : n = 1, 2, \ldots\}$.

a. Show that the set A is not compact by constructing an open cover of A that does not have a finite subcover of A.

2. Suppose $\{p_n\}$ is a convergent sequence in $\mathbb{R}$ with $\lim_{n\to\infty} p_n = p$. Prove, using the definition, that the set $A = \{p\} \cup \{p_n : n \in \mathbb{N}\}$ is a compact subset of $\mathbb{R}$.

3. Show that the set $(0, 1]$ is not compact by constructing an open cover of $(0, 1]$ that does not have a finite subcover of $(0, 1]$.

4. Suppose A and B are compact subsets of $\mathbb{R}$.

b. Prove that $A \cap B$ is compact.

Exercises 4.1 (page 128): Problems:

1. b. $\lim_{x\to-2} 3x + 5 = -1$.

d. $\lim_{x\to-1} 2x^2 - 3x - 4 = 1$.

e. $\lim_{x\to-1} \frac{x^3+1}{x+1} = 3$.

f. $\lim_{x\to 2} \frac{x^3-2x-4}{x^2-4} = \frac{5}{2}$.

7. b. Suppose $f : E \Longrightarrow \mathbb{R}$, p is a limit point of E, and $\lim_{x\to p} f(x) = L$. If $f(x) \geq 0 \ \forall x \in E$, prove that $\lim_{x\to p} \sqrt{f(x)} = \sqrt{L}$.

11. Suppose $E \subset \mathbb{R}$, $f, g : E \Longrightarrow \mathbb{R}$, and p is a limit point of E.

If $\lim_{x\to p} f(x) = A$ and $\lim_{x\to p} g(x) = B$, then

a. $\lim_{x\to p} [f(x) + g(x)] = A + B$.

b. $\lim_{x\to p} [f(x)g(x)] = AB$.

1. Use the $\epsilon - \delta$ definition of limit to prove that $\lim_{x\to 0} \frac{x}{|x|} \neq -1$.

2. Let

$$f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ -x & \text{if } x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$

Prove that $\lim_{x\to p} f(x)$ exists if and only if $p = 0$.