Homework #5

Math 553 DUE: November 10, 2008

`Exercises 2.4 (page 72): Problems:`

7. Determine the limit points and the isolated points of each of the following sets.
d. $\mathbb{N}$

`Exercises 3.1 (page 100): Problems:`

1. Prove Theorem 3.1.5 : Every open interval in $\mathbb{R}$ is an open subset of $\mathbb{R}$.

2. Show that every finite subset of $\mathbb{R}$ is closed.

3. Show that the intervals $(-\infty, a]$ and $[a, \infty]$ are closed subsets of $\mathbb{R}$.

5 a. Let $F$ be a closed subset of $\mathbb{R}$ and let $\{p_n\}$ be a sequence in $F$ which converges to $p \in \mathbb{R}$. Prove that $p \in F$.

6. a. Prove Theorem 3.1.6 (a) For any collection $\{O_\alpha : \alpha \in I\}$ of open subsets of $\mathbb{R}$, $\cup_{\alpha \in I} O_\alpha$ is open.

b. Give an example of an infinite collection $\{F_n\}_{n=1}^{\infty}$ of closed subsets of $\mathbb{R}$ such that $\cup_{n=1}^{\infty} F_n$ is not closed.

10. Prove that the set of limit points of a set is closed.

1. Give an example of an infinite collection $\{O_n\}_{n=1}^{\infty}$ of open subsets of $\mathbb{R}$ such that $\cap_{n=1}^{\infty} O_n$ is not open.

2. Let E be a subset of $\mathbb{R}$ and $p \in \mathbb{R}$. If there exists a sequence of points in $E \setminus \{p\}$ that converges to $p$, then $p$ is a limit point of $E$.

3. Let $E$ be a closed and bounded set. Prove that $E$ contains its infimum and supremum.

4. Suppose $A$ is an open set and $B$ is a closed set. Prove that $A \setminus B$ is an open set and $B \setminus A$ is a closed set.