Homework #4

Math 553 DUE: November 3, 2009

`Exercises 2.4 (page 72): Problems:`

3. Find all the subsequential limits of the following sequences.
b. $\{\cos \frac{n\pi}{2}\}$
f. $\{(1.5 + (-1)^n)^n\}$.

7. Determine the limit points and the isolated points of each of the following sets.
c. $(0, 1) \cup \{2\}$
d. $\mathbb{N}$
e. $\mathbb{R} \setminus \mathbb{Q}$
f. $\mathbb{Q} \cap (0, 1)$

8. Let A be a nonempty subset of $\mathbb{R}$ that is bounded above and let $\alpha = \sup A$. If $\alpha \notin A$, prove that α is a limit point of A.

`Exercises 2.6 (page 85): Problems:`

1. If $\{a_n\}$ and $\{b_n\}$ are Cauchy sequences in $\mathbb{R}$, prove (without using Theorem 2.6.4) that $\{a_n + b_n\}$ and $\{a_n b_n\}$ are Cauchy.

1. Determine three subsequences, of the sequence $\{\sin \frac{\pi\sqrt{n}}{2}\}$, each converging to a different limit.

2. Use only the definition to show that the following sequences are or are not Cauchy.
   (a) $\{\frac{2^n-1}{2^n}\}$
   (b) $\{\frac{2n^2+1}{n^2}\}$
3. Use the definition to prove that the sequence $\{\frac{1}{n^2}\}$ is Cauchy.
4. Use the definition to prove that the sequence $\{(-1)^n\}$ is not a Cauchy sequence.
5. Use the definition to prove that the sequence $\{n\}$ is not a Cauchy sequence.
6. If $\{a_n\}$ has two subsequences that converge to different limits, then prove that $\{a_n\}$ diverges.
7. Let A be a nonempty subset of $\mathbb{R}$ that is bounded below and let $\beta = \inf A$. If $\beta \notin A$, prove that β is a limit point of A.