Homework #3

Math 553 DUE: October 27, 2009

1. If the sequence $\{a_n\}$ is decreasing and bounded below then prove that $\{a_n\}$ converges.
2. Use the definition to prove that the following are true.
   (a) $\lim_{n\to\infty} \sqrt{\frac{n^4+1}{25}} = +\infty$
   (b) $\lim_{n\to\infty} \frac{n^3+11}{n^2+65} = +\infty$
   (c) $\lim_{n\to\infty} (n^5 - n^6) = -\infty$
   (d) $\lim_{n\to\infty} \frac{n^3-1}{n^2} = \infty$
   (e) $\lim_{n\to\infty} (n^2 + (-1)^n n) = \infty$
3. If the sequence $\{a_n\}$ is increasing and not bounded above, then prove that $\{a_n\}$ diverges to ∞.
4. If $a_n \longrightarrow -\infty$ and $\{b_n\}$ converges, then prove that $\{a_n + b_n\}$ diverges to $-\infty$.
5. If $a_n > 0$ for all n and $\{a_n\}$ converges to 0, then prove that $\{\frac{1}{a_n}\}$ diverges to ∞.

`Exercises 2.3 (page 66): Problems:`

7. For each of the following, prove that the sequence $\{a_n\}$ converges and find the limit.
a. $\{a_{n+1}\} = \frac{1}{6}(2a_n + 5)$, $a_1 = 2$.