Homework #2

Math 553 DUE: October 8, 2009

1. Prove using the $\epsilon - n_0$ definition that the following sequences converge.
   (a) $\{\frac{6n-2}{5n-7}\}$.
   (b) $\{\frac{80}{\sqrt{5n}}\}$.
   (c) $\{\frac{\cos n}{n^2+17}\}$.
2. Suppose that $x_n \longrightarrow a$ and $c \in \mathbb{R}$. Prove, using the $\epsilon - n_0$ definition, that $cx_n \longrightarrow ca$:
3. Suppose that $x_n \longrightarrow a$ and $y_n \longrightarrow b$. Prove, using the $\epsilon - n_0$ definition, that $x_n - y_n \longrightarrow a - b$:
4. Prove that if $x_n \longrightarrow a$ and $|y_n - x_n| \le \frac{1}{n}$ for all n, then $y_n \longrightarrow a$.
5. (a) Prove that if $x_n \longrightarrow 0$ and $x_n \ge 0$ for all n, then $\sqrt{x_n} \longrightarrow 0$.

   (b) Prove that if $x_n \longrightarrow a > 0$ and $x_n \ge 0$ for all n, then $\sqrt{x_n} \longrightarrow \sqrt{a}$.
6. Prove that if $x_n \longrightarrow a$ and $a < c$, then there exists an n_0 such that $x_n < c$ for all $n \ge n_0$.

`Exercises 2.1 (page 52):`
`Problems:`

8. For each of the following sequences, prove, using an $\epsilon - n_0$ argument that the sequence converges to the given limit a; that is, given $\epsilon > 0$, determine n_0 such that $|a_n - a| < \epsilon\ \forall n \ge n_0$.
b. $\{\frac{2n+5}{6n-3}\}$, $a = \frac{1}{3}$
d. $\{1 - \frac{(-1)^n}{n}\}$, $a = 1$
e. $\{\frac{(-1)^n n}{n^2+1}\}$, $a = 0$

13. Let $\{a_n\}$ be a sequence in $\mathbb{R}$ with $\lim_{n\to\infty} a_n = a$. Prove that $\lim_{n\to\infty} (a_n)^3 = a^3$

15. Prove that if $\{a_n\}$ converges to a, then $\{|a_n|\}$ converges to $|a|$. Is the converse true?

`Exercises 2.2 (page 59):`
`Problems:`

1. Prove
Theorem 2.2.1(a): If $\{a_n\}$ and $\{b_n\}$ are convergent sequences of real numbers with $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then $\lim_{n\to\infty} (a_n + b_n) = a + b$

2. Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers.
a. If $\{a_n\}$ and $\{a_n + b_n\}$ both converge, prove that the sequence $\{b_n\}$ converges.

b. Suppose $b_n \neq 0\ \forall n \in \mathbb{N}$. If $\{b_n\}$ and $\{\frac{a_n}{b_n}\}$ both converge, prove that the sequence $\{a_n\}$ also converges.

3. Prove
Theorem 2.2.3: Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers. If $\{b_n\}$ is bounded and $\lim_{n\to\infty} a_n = 0$, then $\lim_{n\to\infty} a_n b_n = 0$