

Exercises 1.4 (page 26):

Problems:

5. Find the supremum and infimum of each of the following sets.

d. $D = \{\cos \frac{n\pi}{4} : n \in \mathbb{N}\}$

h. $H = \{(\frac{3}{2} + (-1)^n)^n : n \in \mathbb{N}\}$

j. $J = \{x^2 : -2 < x < 2\}$

8. Let A be a nonempty subset of \mathbb{R} . If $\alpha = \sup A$ is finite, then prove that for each $\epsilon > 0$, there is an $a \in A$ such that $\alpha - \epsilon < a \leq \alpha$.

11. Use the least upper bound property of \mathbb{R} to prove that every nonempty subset of \mathbb{R} that is bounded below has an infimum.

Exercises 1.5 (page 30):

Problems:

3. If $r \neq 0$ is a rational number and x is an irrational number, then prove that $r+x$ and rx are irrational.

6 a. Prove that between any two rational numbers there exists an irrational number.

6 b. Prove that between any two real numbers there exists an irrational number.

7. If $x > 0$, then prove that there exists an $n \in \mathbb{N}$ such that $\frac{1}{2^n} < x$.