Homework #3

Math 546 DUE: February 13, 2012

`EXERCISES (page 287):`

`Problems:`

6. Show that the correspondence $x \longrightarrow 3x$ from $\mathbb{Z}_4$ to $\mathbb{Z}_{12}$ does not preserve multiplication.

8. Prove that every ring homomorphism $\phi$ from $\mathbb{Z}_n$ to itself has the form $\phi(x) = ax$, where $a^2 = a$.

12. Let $\mathbb{Z}_3[i] = \{a + bi | a, b \in \mathbb{Z}_3\}$ (see Example 9 in Chapter 13). Show that the field $\mathbb{Z}_3[i]$ is ring-isomorphic to the field $\mathbb{Z}_3[x]/ < x^2 + 1 >$.

14. Let $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} | a, b \in \mathbb{Z}\}$. Let $H = \{\begin{bmatrix} a & 2b \\ b & a \end{bmatrix} | a, b \in \mathbb{Z}\}$.
Show that $\mathbb{Z}[\sqrt{2}]$ and $H$ are isomorphic as rings.

16. Let $R = \{\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} | a, b \in \mathbb{Z}\}$. Prove or disprove that the mapping $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \longrightarrow a$ is a homomorphism.

18. Is the mapping from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{10}$ given by $x \longrightarrow 2x$ a ring homomorphism?

21. Determine all ring homomorphisms from $\mathbb{Z}_6$ to $\mathbb{Z}_6$. Determine all ring homomorphisms from $\mathbb{Z}_{20}$ to $\mathbb{Z}_{30}$.

22. Determine all ring isomorphisms from $\mathbb{Z}_n$ to itself.