Homework #2

Math 546 DUE: January 25, 2012

`EXERCISES (page 255):`

`Problems:`

26. Let d be a positive integer. Prove that $\mathbb{Q}[\sqrt{d}] = \{a + b\sqrt{d} | a, b \in \mathbb{Q}\}$ is a field.

38. Construct a multiplication table for $\mathbb{Z}_2[i]$, the ring of Gaussian integers modulo 2. Is this ring a field? Is it an integral domain?

42. Suppose that a and b belong to a commutative ring and ab is a zero divisor. Show that either a or b is a zero-divisor.

44. Suppose that R is a commutative ring without zero-divisors. Show that the characteristic of R is 0 or prime.

`EXERCISES (page 269):`

`Problems:`

9. If n is an integer greater than 1, show that $< n >= n\mathbb{Z}$ is a prime ideal of $\mathbb{Z}$ if and only if n is prime.

10. If A and B are ideals of a ring, show that the sum of A and B, $A + B = \{a + b | a \in A$ and $b \in B\}$, is an ideal.

14. Let A and B be ideals of a ring. Prove that $AB \subseteq A \cap B$.

1. What can be said about the characteristic of a ring $R \neq 0$ in which $x = -x \ \forall x \in R$?

2. Establish the following assertions concerning the characteristic of a ring R:
   (a) if char $R > 0$, then char $S \leq$ char R for any subring S of R.
   (b) if R is an integral domain and S is a subdomain of R, then char $S =$ char R.