Homework #1

Math 546 DUE: January 18, 2012

EXERCISES (page 242):

Problems:

2. The set $\{0, 2, 4, 6, 8\}$ under addition and multiplication modulo 10 has a unity. Find it.

14. Let a and b belong to a ring R and let m be an integer. Prove that $m.(ab) = (m.a)b = a(m.b)$.

22. Let R be a commutative ring with unity and let $\mathrm{U}(R)$ denote the set of units of R. Prove that $\mathrm{U}(R)$ is a group under the multiplication of R. (This group is called the **group of units** of R.

40. Let $M_2(\mathbb{Z})$ be the ring of all 2×2 matrices over the integers and let $R = \{\begin{bmatrix} a & a+b \\ a+b & b \end{bmatrix} | a, b \in \mathbb{Z}\}$. Prove or disprove that R is a subring of $M_2(\mathbb{Z})$.

40. Let $R = \{\begin{bmatrix} a & a \\ b & b \end{bmatrix} | a, b \in \mathbb{Z}\}$. Prove or disprove that R is a subring of $M_2(\mathbb{Z})$.

44. Suppose there is a positive even integer n such that $a^n = a$ for all elements of some ring. Show that $-a = a$ for all a in the ring.

46. Show that $2\mathbb{Z} \cup 3\mathbb{Z}$ is not a subring of $\mathbb{Z}$.

50. Suppose that R is a ring such that $a^2 = a$ for all a in R. Show that R is commutative.