QUIZ #3

Math 531 February 6, 2012

KEY

Answer the following. Show your work for full credit.

1. (5 points) Determine whether the linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ defined by $T(a_1, a_2, a_3) = (a_1 + a_2 + a_3, a_3, a_2)$ is invertible? Justify your answer.

   $T(a_1, a_2, a_3) = (0, 0, 0) \iff (a_1 + a_2 + a_3, a_3, a_2) = (0, 0, 0) \iff a_1 + a_2 + a_3 = 0, a_3 = 0, a_2 = 0$ $\iff a_1 = 0, a_2 = 0, a_3 = 0$. Hence $N(T) = \{(0, 0, 0)\}$. Thus, T is 1-1 by Theorem 2.4 (page 71). Now, by Theorem 2.5 (page 71), T is onto. Therefore, T is invertible.

2. (5 points) Let V and W be vector spaces of dimension n and let $T : V \longrightarrow W$ be linear. If T is 1-1, then prove that T is onto.
   [Hint : dim(V) = nullity(T) + rank(T).]
   Assume T is 1-1. By Theorem 2.4 (page 71) $N(T) = \{0_V\}$. So, nullity(T)=0. By Theorem 2.3, nullity(T)+rank(T)=dim(V). Then 0+rank(T)=dim(V) $\implies$ rank(T)=dim(V). But dim(V)=dim(W). So rank(T)=dim(W). Now R(T) is a subspace of W and dim(R(T))= rank(T)=dim(W). Hence R(T)=W and T is onto.