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1. Find the volume above the xy-plane bounded by the paraboloid $z = x^2 + y^2$ and the planes $x = \pm 1$, $y = \pm 1$.

Ans. $\int_{-1}^1 [\int_{-1}^1 (x^2 + y^2) dx] dy$

2. Find the volume above the xy-plane bounded by the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 2$.

Ans. $\int_{-1}^1 [\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (2 - x - y) dy] dx$

3. Find the volume above the xy-plane bounded by the cylinder $y = 4 - x^2$ and the planes $y = 3x$ and $z = x + 4$.

Ans. $\int_{-4}^1 [\int_{3x}^{4-x^2} (x + 4) dy] dx$

4. Find the volume of the solid bounded by the coordinate planes, the planes $x = 2$ and $y = 5$, and the surface $2z = xy$.

Ans. $\int_0^2 [\int_0^5 (\frac{xy}{2}) dy] dx$

5. Find the volume above the xy-plane bounded by the cylinder $x^2 + y^2 = 9$ and the paraboloid $3z = x^2 + y^2$.

Ans. $\int_{-3}^3 [\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} ((\frac{x^2+y^2}{3})) dy] dx$

6. Find the volume of the solid in the first octant bounded by the cylinder $4y = x^2$ and the planes $x = 0$, $z = 0$, $y = 4$, and $x - y + 2z = 2$.

Ans. $\int_0^4 [\int_{\frac{x^2}{4}}^4 (\frac{2-x-y}{2}) dy] dx$

7. Change the order of integration of

(a) $\int_0^1 [\int_y^1 \frac{1}{1+x^4} dx] dy.$

Ans. $\int_0^1 [\int_0^x \frac{1}{1+x^4} dy] dx$

(b) $\int_0^3 [\int_{x^2}^{2x+3} x dy] dx.$

Ans. $\int_0^3 [\int_0^{\sqrt{y}} x dx] dy + \int_3^9 [\int_{\frac{y-3}{2}}^{\sqrt{y}} x dy] dx$

(c) $\int_0^1 [\int_{x^2}^x (2x + 2y) dy] dx.$

Ans. $\int_0^1 [\int_y^{\sqrt{y}} (2x + 2y) dx] dy$

(d) $\int_0^4 [\int_0^y 3\sqrt{y^2 + 9} dx] dy.$

Ans. $\int_0^4 [\int_x^4 3\sqrt{y^2 + 9} dx] dy$

(e) $\int_1^2 [\int_{y^2}^{y^3} dx] dy.$

Ans. $\int_1^4 [\int_{\sqrt[3]{x}}^{\sqrt{x}} dy] dx + \int_4^8 [\int_{\sqrt[3]{x}}^2 dy] dx.$

(f) $\int_0^{\frac{\pi}{2}} [\int_0^{\cos x} y dy] dx.$

Ans. $\int_0^1 [\int_0^{\cos^{-1} y} y dx] dy$

(g) $\int_1^{e^3} [\int_0^{\frac{1}{y}} e^{xy} dx] dy.$

Ans. $\int_0^{\frac{1}{e^3}} [\int_1^{e^3} e^{xy} dy] dx + \int_{\frac{1}{e^3}}^1 [\int_1^{\frac{1}{x}} e^{xy} dy] dx$

(h) $\int_1^3 [\int_0^{\ln y} ye^x dx] dy.$

Ans. $\int_0^{\ln 3} [\int_{e^x}^3 ye^x dy] dx$

- (i) $\int_0^1 [\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} y dx] dy$.
 Ans. $\int_{-1}^1 [\int_0^{\sqrt{1-x^2}} y dy] dx$.
- (j) $\int_0^\pi [\int_0^x x \cos y dy] dx$.
 Ans. $\int_0^\pi [\int_y^\pi x \cos y dx] dy$
- (k) $\int_{-3}^2 [\int_0^{y^2} (x^2 + y) dx] dy$.
 Ans. $\int_0^4 [\int_{\sqrt{x}}^2 (x^2 + y) dy] dx + \int_0^9 [\int_{-3}^{-\sqrt{x}} (x^2 + y) dy] dx$.
- (l) $\int_0^2 [\int_{\frac{y}{2}}^1 (e^{x^3} y) dx] dy$.
 Ans. $\int_0^1 [\int_0^{2x} (e^{x^3} y) dy] dx$.
- (m) $\int_0^1 [\int_{1-y}^1 (x + y^2) dx] dy$.
 Ans. $\int_0^1 [\int_{1-x}^1 (x + y^2) dy] dx$.
- (n) $\int_{-1}^1 [\int_{|y|}^1 (x + y)^2 dx] dy$.
 Ans. $\int_0^1 [\int_0^x (x + y)^2 dy] dx + \int_0^1 [\int_{-x}^0 (x + y)^2 dy] dx$.

8. Use double integrals to find the area of the region bounded by the given curves and lines.

- (a) The parabola $x = y^2$ and the line $y = x - 2$.
 Ans. $\int_{-1}^2 [\int_{y^2}^{y+2} dx] dy$
- (b) The parabola $y = x - x^2$ and the line $x + y = 0$.
 Ans. $\int_0^2 [\int_{-x}^{x-x^2} dy] dx$
- (c) The axes and the line $2x + y = 2a$ ($a > 0$).
 Ans. $\int_0^a [\int_0^{2a-2x} dy] dx$
- (d) The y-axis, the line $y = 3x$, and the line $y = 6$.
 Ans. $\int_0^2 [\int_{3x}^6 dy] dx$
- (e) The x-axis, the curve $y = e^{-x}$, and the lines $x = 0$, $x = a$ ($a > 0$).
 Ans. $\int_0^a [\int_0^{e^{-x}} dy] dx$
- (f) The parabolas $y = x^2$ and $y = 2x - x^2$.
 Ans. $\int_0^1 [\int_{x^2}^{2x-x^2} dy] dx$
9. (a) Evaluate $\int \int_D (x - y^2) dx dy$, where D is the region $\{(x, y) | 0 \leq x \leq 4, \sqrt{x} \leq y \leq 2\}$.
 Ans. $\int_0^4 [\int_{\sqrt{x}}^2 (x - y^2) dy] dx$
- (b) Evaluate $\int \int_D (1 - \sin \pi x) y dx dy$, where D is the region $\{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq x\}$.
 Ans. $\int_0^1 [\int_0^x (1 - \sin \pi x) y dy] dx$
- (c) Evaluate $\int \int_D (x^2 + y) dx dy$, where D is the region $\{(x, y) | -3 \leq y \leq 2, 0 \leq x \leq y^2\}$.
 Ans. $\int_{-3}^2 [\int_0^{y^2} (x^2 + y) dx] dy$
- (d) Evaluate $\int \int_D (x^2 + y^2) dx dy$, where D is the region bounded by the positive x-axis, the positive y-axis, and the line $3x + y = 9$.
 Ans. $\int_0^3 [\int_0^{9-3x} (x^2 + y^2) dy] dx$
- (e) Evaluate $\int \int_D y^2 \sqrt{x} dA$, where D is the region $\{(x, y) | x > 0, x^2 < y < 10 - x^2\}$.
 Ans. $\int_0^{\sqrt{5}} [\int_{x^2}^{10-x^2} y^2 \sqrt{x} dy] dx$

10. Evaluate $\int \int \int_B dx dy dz$, where B is the region bounded by the coordinate planes and the plane $x + y + z = 1$.

Ans. $\int_0^1 [\int_0^{1-x} (\int_0^{1-x-y} dz) dy] dx$

11. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

Ans. $\int_{-a}^a [\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} (\int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} dz) dy] dx$

12. Evaluate

(a) $\int_0^1 \int_0^{x^2} \int_0^{xy^3} 18x^3y^2z dz dy dx$
 Ans. $\frac{1}{24}$

(b) $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x dz dx dy$
 Ans. $\frac{4}{35}$

(c) $\int_0^2 \int_0^\pi \int_0^{\ln 4} x^3 \cos \frac{y}{2} e^z dz dy dx$
 Ans. 24

(d) $\int_0^1 \int_0^{\sqrt{3z}} \int_0^{\sqrt{3(y^2+z^2)}} xyz \sqrt{x^2 + y^2 + z^2} dx dy dz$
 Ans. $\frac{31}{15}$

(e) $\int_0^{\sqrt{\frac{\pi}{2}}} \int_x^{\sqrt{\frac{\pi}{2}}} \int_1^3 \sin y^2 dz dy dx$
 Ans. $2 \int_0^{\sqrt{\frac{\pi}{2}}} \int_0^y \sin y^2 dx dy = 1$

13. Evaluate $\int \int \int_W z dx dy dz$, where W is the region bounded by the four planes $x = 0$, $y = 0$, $z = 0$, $z = 1$, and the cylinder $x^2 + y^2 = 1$, with $x \geq 0$, $y \geq 0$.

Ans. $\int_0^1 [\int_0^{\sqrt{1-x^2}} (\int_0^1 z dz) dy] dx$

14. Evaluate $\int \int \int_W z e^{x+y} dx dy dz$, where $W = [0, 1] \times [0, 1] \times [0, 1]$.

Ans. $\frac{(e-1)^2}{2}$

15. Evaluate $\int \int \int_W (x^2 + y^2 + z^2) dx dy dz$, where W is the region bounded by $x + y + z = a$ ($a > 0$), $x = 0$, $y = 0$, and $z = 0$.

Ans. $\int_0^a [\int_0^{a-x} (\int_0^{a-x-y} (x^2 + y^2 + z^2) dz) dy] dx = \frac{a^5}{20}$

16. Set up a triple integral for the volume of each of the following solid regions.

(a) The region in the first octant bounded above by the cylinder $z = 1 - y^2$ and lying between the vertical planes given by $x + y = 1$ and $x + y = 3$

Ans. $\int_0^1 [\int_{1-y}^3 (\int_0^{1-y^2} dz) dx] dy$

(b) The upper hemisphere given by $z = \sqrt{1 - x^2 - y^2}$.

Ans. $\int_{-1}^1 [\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (\int_0^{\sqrt{1-x^2-y^2}} dz) dy] dx$

(c) The region bounded below by the paraboloid $z = x^2 + y^2$ and above by the sphere $x^2 + y^2 + z^2 = 6$.

Ans. $\int_{-\sqrt{2}}^{\sqrt{2}} [\int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} (\int_{x^2+y^2}^{\sqrt{6-x^2-y^2}} dz) dy] dx$

17. Use the triple integration to find the volumes of the given regions.

(a) The region in the first octant bounded by the cylinder $x = 4 - y^2$ and the planes $y = z$, $x = 0$, $z = 0$.

Ans. $\int_0^2 [\int_0^{4-y^2} (\int_0^y dz) dx] dy$

(b) The region above the xy-plane bounded by the surfaces $z^2 = 16y$, $z^2 = y$, $y = x$, $y = 4$, and $x = 0$.

Ans. $\int_0^4 [\int_0^y (\int_{\sqrt{y}}^{4\sqrt{y}} dz) dx] dy$

- (c) The region bounded by the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + 3y^2$.
 Ans. $\int_{-\sqrt{2}}^{\sqrt{2}} [\int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} (\int_{x^2+y^2}^{8-x^2-y^2} dz) dx] dy$
- (d) The region bounded by the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
 Ans. $\int_{-b}^b [\int_{-a\sqrt{1-\frac{y^2}{b^2}}}^{a\sqrt{1-\frac{y^2}{b^2}}} (\int_{-c\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}}^{c\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}} dz) dx] dy$
- (e) The region bounded by the cylinder $z = 4 - y^2$ and the paraboloid $z = x^2 + 3y^2$.
 Ans. $\int_{-1}^1 [\int_{-2\sqrt{1-y^2}}^{2\sqrt{1-y^2}} (\int_{x^2+3y^2}^{4-y^2} dz) dy] dx$
- (f) The region bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
 Ans. $\int_0^b [\int_0^{a(1-\frac{y}{b})} (\int_0^{1-\frac{x}{a}-\frac{y}{b}} dz) dx] dy$
- (g) The region bounded by the cylinder $x^2 + y^2 = 4x$, the xy-plane and the paraboloid $4z = x^2 + y^2$.
 Ans. $\int_0^4 [\int_{-\sqrt{4x-x^2}}^{\sqrt{4x-x^2}} (\int_0^{\frac{1}{4}(x^2+y^2)} dz) dy] dx$

18. Use a double integrals in polar coordinates to find the areas of the indicated regions.

- (a) The cardioid $r = a(1 + \cos \theta)$ ($a > 0$).
 Ans. $2 \int_0^\pi [\int_0^{a(1+\cos \theta)} r dr] d\theta = \frac{3\pi a^2}{2}$
- (b) The circle $r = a$.
 Ans. $\int_0^{2\pi} [\int_0^a r dr] d\theta = \pi a^2$
- (c) The circle $r = 2a \sin \theta$.
 Ans. $\int_0^\pi [\int_0^{2a \sin \theta} r dr] d\theta = \pi a^2$
- (d) The circle $r = 2a \cos \theta$.
 Ans. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [\int_0^{2a \cos \theta} r dr] d\theta = \pi a^2$
- (e) One loop of $r = a \cos 2\theta$ ($a > 0$).
 Ans. $2 \int_0^{\frac{\pi}{4}} [\int_0^{a \cos 2\theta} r dr] d\theta = \frac{\pi a^2}{8}$
- (f) One loop of $r = 3 \cos 3\theta$.
 Ans. $2 \int_0^{\frac{\pi}{6}} [\int_0^{3 \cos 3\theta} r dr] d\theta = \frac{3\pi}{4}$
- (g) The region inside the lemniscate $r^2 = 2a^2 \cos 2\theta$ and outside the circle $r = a$ ($a > 0$).
 Ans. $4 \int_0^{\frac{\pi}{6}} [\int_a^{\sqrt{2} \cos 2\theta} r dr] d\theta = \frac{a^2}{3} (3\sqrt{3} - \pi)$
- (h) The region inside $r = \tan \theta$ and between $\theta = 0$ and $\theta = \frac{\pi}{4}$.
 Ans. $\int_0^{\frac{\pi}{4}} [\int_0^{\tan \theta} r dr] d\theta = \frac{1}{8} (4 - \pi)$
- (i) The region inside the cardioid $r = 2a(1 + \cos \theta)$ and outside the circle $r = 3a$ ($a > 0$).
 Ans. $2 \int_0^{\frac{\pi}{3}} [\int_{3a}^{2a(1+\cos \theta)} r dr] d\theta = \frac{a^2}{2} (9\sqrt{3} - 2\pi)$
- (j) The region inside the cardioid $r = 1 + \cos \theta$ and to the right of the line $x = \frac{3}{4}$.
 Ans. $2 \int_0^{\frac{\pi}{3}} [\int_{\frac{3}{4} \sec \theta}^{1+\cos \theta} r dr] d\theta = \frac{1}{16} (8\pi + 9\sqrt{3})$

19. Write the integral in the form $\int_\alpha^\beta \int_{r_1(\theta)}^{r_2(\theta)} z r dr d\theta$.

- (a) $\int_0^2 \int_0^{\sqrt{4-x^2}} z dy dx$.
 Ans. $\int_0^{\frac{\pi}{2}} [\int_0^2 z r dr] d\theta$
- (b) $\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} z dy dx$.
 Ans. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [\int_0^3 z r dr] d\theta$
- (c) $\int_{-1}^0 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} z dx dy$.
 Ans. $\int_\pi^{2\pi} [\int_0^1 z r dr] d\theta$

$$(d) \int_0^1 \int_{x^2}^x z dy dx.$$

$$\text{Ans. } \int_0^{\frac{\pi}{4}} \left[\int_0^{\sec \theta \tan \theta} z r dr \right] d\theta$$

$$(e) \int_0^4 \int_0^{\sqrt{4-(x-2)^2}} z dy dx.$$

$$\text{Ans. } \int_0^{\frac{\pi}{2}} \left[\int_0^{4 \cos \theta} z r dr \right] d\theta$$

$$(f) \int_0^2 \int_0^{\sqrt{2y-y^2}} z dx dy.$$

$$\text{Ans. } \int_0^{\frac{\pi}{2}} \left[\int_0^{2 \sin \theta} z r dr \right] d\theta$$