Math 252 Sample Quiz #6 Winter 2011

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1. Find the surface area of the unit sphere given by $\mathbf{r}(u,v) = \sin u \cos v\mathbf{i} + \sin u \sin v\mathbf{j} + \cos u\mathbf{k}$, where the domain D is given by $0 \le u \le \pi$, $0 \le v \le 2\pi$. ($|\mathbf{r_u} \times \mathbf{r_v}| = |\sin^2 u \cos v\mathbf{i} + \sin^2 u \sin v\mathbf{j} + \sin u \cos u\mathbf{k}| = \sin u$ (Answer: 4π))

2. Find the surface area of the torus given by $\mathbf{r}(u,v) = (2+\cos u)\cos v\mathbf{i} + (2+\cos u)\sin v\mathbf{j} + \sin u\mathbf{k}$, where the domain D is given by $0 \le u \le 2\pi$, $0 \le v \le \pi$. (($|\mathbf{r_u} \times \mathbf{r_v}| = 2 + \cos u$ (Answer; $8\pi^2$))

3. Find the surface area of the portion of sphere, center (0,0), of radius 4 that lies inside the cylinder $x^2 + y^2 = 12$ and above the xy-plane.
$\int_0^{2\pi} \int_0^{\frac{\pi}{3}} 16 \sin\phi d\phi d\theta = 16\pi$.

4. Let $\Phi(u,v) = (u-v, u+v, uv)$ and let D be a unit disk in the uv plane. Find the area of $\Phi(D)$.

5. Find the area of the portion of the unit sphere that is cut out by the cone $z \ge \sqrt{x^2+y^2}$.

6. Find the area of the surface defined by $x + y + z = 1$, $x^2 + 2y^2 \le 1$.

7. Find the area of the graph of the function $f(x,y) = \frac{2}{3}(x^{\frac{3}{2}} + y^{\frac{3}{2}})$ that lies over the domain $[0,1] \times [0,1]$.

8. Express the surface area of the following graphs over the indicated region D as a double integral. Do not evaluate.

   (a) $(x+2y)^2$; $D = [-1,2] \times [0,2]$

   (b) $xy + \frac{x}{y+1}$; $D = [1,4] \times [1,2]$

   (c) $xy^3 e^{x^2y^2}$; D = unit circle centered at the origin

   (d) $y^3 \cos^2 x$; D = triangle with vertices $(-1,1), (0,2)$, and $(1,1)$.

9. Evaluate the surface integral $\int\int_S (y^2 + 2yz) dS$, where S is the first-octant portion of the plane $2x + y + 2z = 6$.
$3\int_0^3 \int_0^{2(3-x)} y(3-x) dy dx = \frac{243}{2}$.

10. Evaluate $\int\int_S 6xy dS$,where S is the first-octant portion of the plane $x+y+z = 1$. $6\sqrt{3}\int_0^1 \int_0^{1-y} (y - y^2 - zy) dz dy = \frac{\sqrt{3}}{4}$.

11. Evaluate $\int\int_S (x^2 + y^2) dS$, where S is the part of the surface of the paraboloid $z = f(x,y) = 1 - x^2 - y^2$ that lies above the xy-plane.
$\int_0^{2\pi} \int_0^1 r^2 \sqrt{4r^2+1} r dr d\theta$.

12. Evaluate $\int\int_S z^2 dS$, where S is the portion of the cone $z = \sqrt{x^2+y^2}$ for which $1 \le x^2 + y^2 \le 4$.
$\sqrt{2}\int_0^{2\pi} \int_1^2 r^3 dr d\theta$.

13. Evaluate $\int\int_S (x+y+z) dS$, where S is the portion of the plane $x + y = 1$ in the first octant for which $0 \le z \le 1$.
$\sqrt{2}\int_0^1 \int_0^1 (1+z) dx dz = \frac{3\sqrt{2}}{2}$.

14. Evaluate the surface integral $\int\int_S (x+z) dS$, where S is the first-octant portion of the cylinder $y^2 + z^2 = 9$ between $x = 0$ and $x = 4$.
In parametric form the surface is given by $\mathbf{r}(x,\theta) = x\mathbf{i} + 3\cos\theta\mathbf{j} + 3\sin\theta\mathbf{k}$, $0 \le x \le 4$, $0 \le \theta \le \frac{\pi}{2}$.
$\| \mathbf{r_x} \times \mathbf{r_\theta} \| = 3$. $\int_0^4 \int_0^{\frac{\pi}{2}} (3x + 9\sin\theta) d\theta dx = 12\pi + 4$.

15. Evaluate the surface integral $\int\int_S z dS$, where S is the upper half of a sphere of radius 2. $\mathbf{r}(\theta,\phi) = 2\sin\phi\cos\theta\mathbf{i} + 2\sin\phi\sin\theta\mathbf{j} + 2\cos\phi\mathbf{k}$. $\| \mathbf{r}_\theta \times \mathbf{r}_\phi \| = 4\sin\phi$. $\int_0^{2\pi}\int_0^{\frac{\pi}{2}}(4\sin 2\phi)d\phi d\theta = 8\pi$.

16. Evaluate $\int\int_S y dS$, where S is the portion of the cylinder $x^2+y^2=3$ that lies between $z=0$ and $z=6$. $\mathbf{r}(z,\theta) = \sqrt{3}\cos\theta\mathbf{i} + \sqrt{3}\sin\theta\mathbf{j} + z\mathbf{k}$, $0 \le z \le 6$, $0 \le \theta \le 2\pi$. $\| \mathbf{r_z} \times \mathbf{r}_\theta \| = \sqrt{3}$. $\int_0^{2\pi}\int_0^6 \sin\theta dz d\theta = 0$.

17. Evaluate $\int\int_S xyz dS$, where S is the triangle with vertices $(1,0,0), (0,2,0)$, and $(0,1,1)$.

18. Evaluate $\int\int_S z dS$, where S is the upper hemisphere of radius a; that is, the set of (x,y,z) with $z = \sqrt{a^2-x^2-y^2}$.

19. Evaluate $\int\int_S (x+y+z) dS$, where S is the boundary of the unit ball B; that is, S is the set of (x,y,z) with $x^2+y^2+z^2=1$.

20. Compute the area of the portion of the cone $x^2+y^2=z^2$, with $z^2 \ge 0$ that is inside the sphere $x^2+y^2+z^2=2Rz$, where R is a positive constant.
$\| r_\rho \times r_\theta \| = \sqrt{2}\rho$