Math 252 Sample Quiz #4 Winter 2011

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1. Use a triple integral in spherical coordinates to find the volume of the sphere $x^2 + y^2 + z^2 = a^2$.
   Ans. $\int_0^{2\pi} \int_0^{\pi} \int_0^{a} \rho^2 \sin\phi \, d\rho \, d\phi d\theta = \frac{4}{3}\pi a^3$.

2. Use a triple integral in spherical coordinates to find the volume of the sphere $x^2 + y^2 + z^2 = 2z$.
   Ans. $\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^{2\cos\phi} \rho^2 \sin\phi \, d\rho \, d\phi d\theta = \frac{4}{3}\pi$.

3. Find the volume of the region bounded by the sphere $\rho = a$ and the cone $\phi = \alpha$.
   Ans. $\int_0^{2\pi} \int_0^{\alpha} \int_0^{a} \rho^2 \sin\phi \, d\rho \, d\phi d\theta = \frac{4}{3}\pi a^3(1 - \cos\alpha)$.

4. If $0 < b < a$ and $0 < \alpha < \pi$, find the volume of the region bounded by the concentric spheres $\rho = b$, $\rho = a$ and the cone $\phi = \alpha$.
   Ans. $\int_0^{2\pi} \int_0^{\alpha} \int_b^{a} \rho^2 \sin\phi \, d\rho \, d\phi d\theta = \frac{2\pi}{3}(a^3 - b^3)(1 - \cos\alpha)$.

5. Find the volume of the solid region bounded below by the upper nappe of the cone $z^2 = x^2 + y^2$ and above by the sphere $x^2 + y^2 + z^2 = 9$.
   Ans. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{3} \rho^2 \sin\phi \, d\rho \, d\phi d\theta = 9\pi(2 - \sqrt{2})$.

6. Evaluate $\int\int\int_E 16z dx dy$, where E is the upper half of the sphere $x^2 + y^2 + z^2 = 1$.
   Ans. $\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^{1} (16\rho\cos\phi)\rho^2 \sin\phi \, d\rho \, d\phi d\theta = 4\pi$.

7. Convert $\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (x^2 + y^2 + z^2) dz dx dy$ into an integral in

   (a) spherical coordinates. Ans. $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{4}} \int_0^{3\sqrt{2}} (\rho^2)\rho^2 \sin\phi \, d\rho \, d\phi d\theta$.

   (b) cylindrical coordinates. $\int_0^{\frac{\pi}{2}} [\int_0^3 (\int_r^{\sqrt{18-r^2}} (r^2 + z^2) \, dz) r \, dr] \, d\theta$

8. Convert $\int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \int_a^{a+\sqrt{a^2-x^2-y^2}} x dz dy dx$
   into an integral in

   (a) spherical coordinates. Ans. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_{a\sec phi}^{2a\cos\phi} \rho^3 \sin^2\phi \cos\theta d\rho \, d\phi d\theta$.

   (b) cylindrical coordinates. Ans. $\int_0^{2\pi} \int_0^{a} (\int_a^{a+\sqrt{a^2-r^2}} (r cos\theta) \, dz) r \, dr \, d\theta$.

9. Evaluate $\int_C (x - 3y) ds$, where C is the line segment from $(0, 0)$ to $(1, 2)$ and C is parameterized as:

   (a) $x = t$, $y = 2t$, $0 \le t \le 1$.

   (b) $x = \sin t$, $y = 2\sin t$, $0 \le t \le \frac{\pi}{2}$.

10. Evaluate $\int_C (x^2 - y + 3z) ds$, where C is the line segment from $(0, 0, 0)$ to $(1, 2, 1)$. [Answer. $\int_0^1 (t^2 - 2t + 3t)\sqrt{6} dt = \frac{5\sqrt{6}}{6}$].

11. Evaluate $\int_C x ds$, where C is the line segment from $(0, 0)$ to $(1, 1)$ and $y = x^2$ from $(1, 1)$ to $(0, 0)$. [Answer. $\int_0^1 t\sqrt{2} dt + \int_0^1 (1 - t)\sqrt{1 + 4(1 - t)^2} dt = \frac{\sqrt{2}}{2} + \frac{1}{12}(5^{\frac{3}{2}} - 1)$].

12. Evaluate the line integrals $\int_C (x - 3y) dx$, and $\int_C (x - 3y) dy$, if C is the part of the parabola $x = y^2$ that joins the points $(1, 1)$ and $(4, 2)$ [Answer. $\int_1^2 (t^2 - 3t) 2t dt = \frac{-13}{2}$ and $\int_1^2 (t^2 - 3t) dt = \frac{-13}{6}$].

13. Evaluate the line integral $\int_C (y^2 dx - x^2 dy)$ along the two curves given below:

(a) C_1: The parabola $x = t$, $y = t^2$ joining the two points $(0,0)$ and $(2,4)$. [Answer. $\int_0^2 (t^4 - 2t^3)dt = \frac{-8}{5}$].

(b) C_2: The line $x = t$, $y = 2t$ joining the two points $(0,0)$ and $(2,4)$. [Answer. $\int_0^2 2t^2 dt = \frac{16}{3}$].

14. Evaluate the line integral $\int_C xy^2 dx - (x+y)dy$, where C is

(a) The straight line segment from $(0,0)$ and $(1,2)$. [Answer. $\int_0^1 (4t^3 - 6t)dt = -2$].

(b) The parabolic path from $(0,0)$ and $(2,4)$. [Answer. $\int_0^1 (4t^5 - 4t^2 - 8t^3)dt = \frac{-8}{3}$].

(c) The broken line from $(0,0)$ to $(1,0)$ to $(1,2)$. [Answer. $-\int_0^2 (1+t)dt = -4$].

15. Evaluate the line integral $\int_C xy^2 dx - (x+y)dy$, where C is the broken line joining the points $(0,0)$, $(1,1)$, $(2,1)$ in this order.

16. Evaluate the line integral $\int_C \frac{dx}{y} + \frac{dy}{x}$, where C is the part of the hyperbola $xy = 4$ from $(1,4)$ to $(4,1)$. [Answer. $\int_1^4 (\frac{t}{4} - \frac{4}{t^3})dt = 0$].

17. Evaluate the line integral $\int_C xdx + x^2 dy$ from $(-1,0)$ to $(1,0)$.

(a) Along the x-axis. [Answer. $\int_{-1}^1 tdt = 0$].

(b) Along the semicircle $y = \sqrt{1-x^2}$.
$x = -\cos t$, $y = \sin t$, $0 \le t \le \pi$[Answer. $\int_0^\pi (-\cos t)\sin t + (-\cos t)^2 \cos t)dt$].

(c) Along the broken line from $(-1,0)$ to $(0,1)$ to $(1,1)$. [Answer. $\int_{-1}^0 (t+t^2)dt + \int_0^1 tdt - \int_0^1 dt = -\frac{1}{6} + \frac{1}{2} - 1 = -\frac{2}{3}$].

18. Evaluate the line integral $\int_C ydx + (x+2y)dy$ from $(1,0)$ to $(0,1)$, where C is

(a) the arc of the circle $x = \cos t$, $y = \sin t$; [Answer. $\int_0^{\frac{\pi}{2}} (\cos^t - \sin^2 t + 2\sin t \cos t)dt = 1$].

(b) the straight line segment $y = 1 - x$; [Answer. $\int_1^0 (-1)dx = 1$].

(c) the broken line from $(1,0)$ to $(1,1)$ to $(0,1)$. [Answer. $\int_0^1 (1+2y)dy + \int_1^0 dx = 1$].

19. Evaluate $\int_C (3x+4y)dx + (2x+3y^2)dy$, where C is the circle $x^2 + y^2 = 4$ traversed counterclockwise from $(2,0)$. [Answer. $x = 2\cos t, y = 2\sin t$, $0 \le t \le 2\pi$, $\int_0^{2\pi} ((24\sin^2 t - 12\sin t)\cos t + (8 - 24\sin^2 t))dt = -8\pi$].

20. Evaluate $\int_C (x+2)ds$, where C is the curve represented by $\mathbf{c}(t) = t\mathbf{i} + \frac{4}{3}t^{\frac{3}{2}}\mathbf{j} + \frac{1}{2}t^2\mathbf{k}$, $0 \le t \le 6\pi$.

21. (a) Evaluate $\int_S xy^4 ds$, where C is the right half of the circle, $x^2 + y^2 = 1$. [Answer. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (4\cos t)(4\sin t)^4 (4)dt = \frac{8192}{5}$].

(b) Evaluate $\int_S 4x^3 ds$, were C is the line segment from (-2,-1) to (1,2). [Answer. $\int_{-2}^0 4(-2 + 3t)^3 \sqrt{18}dt = -15\sqrt{2}$].

(c) Evaluate $\int_S 4x^3 ds$, were C is the line segment from (1,2) to (-2,-1). [Answer. $\int_0^1 4(1 - 3t)^3 \sqrt{18}dt = -15\sqrt{2}$].

22. Evaluate $\int_C xds$ for each of the following curves. (a) C_1: $y = x^2$, $-1 \le x \le 1$ (b) C_2: The line segment from(-1,1) to (1,1). (c) C_3: The line segment from(1,1) to (-1,1). [Answer: 0 for each part (a)-(c)].

23. Evaluate $\int_C xyzds$, where C is the helix given by, $x = \cos t$, $y = \sin t$, $z = 3t$, $0 \le t \le 4\pi$.

24. Evaluate $\int_C \sin(\pi y)dy + yx^2 dx$, where C is the line segment from (0,2) to (1,4). [Answer. $\frac{7}{6}$].

25. Evaluate $\int_C \sin(\pi y)dy + yx^2 dx$, where C is the line segment from (1,4) to (0,2). [Answer. $-\frac{7}{6}$].

26. Evaluate $\int_C ydx + xdy + zdx$, where C is given by, $x = \cos t$, $y = \sin t$, $z = t^2$, $0 \leq t \leq 2\pi$. [Answer. $\int_0^{2\pi}(-\sin^2 t + \cos^2 t + 2t^3)dt = 8\pi^4$].

27. Evaluate $\int_C (2xydx + x^2dy)$ if:

    (a) C consists of the line segments from $(3,1)$ to $(5,1)$ and from $(5,1)$ to $(5,6)$.

    (b) C is the line segments from $(3,1)$ to $(5,6)$.

    (c) C is the part of the parabola $x = 2t + 1$, $y = 2t^2 - t$, $1 \leq t \leq 2$. [Answer: 141, for (a), (b), (c)]