Math 252 Sample Quiz #2 Winter 2011

EXERCISES
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1. Use double integrals to find the area of the region bounded by the given curves and lines.

   (a) The parabola $x = y^2$ and the line $y = x - 2$.
   Ans. $\int_{-1}^{2}[\int_{y^2}^{y+2} dx]\, dy$

   (b) The parabola $y = x - x^2$ and the line $x + y = 0$.
   Ans. $\int_0^2[\int_{-x}^{x-x^2} dy]\, dx$

   (c) The axes and the line $2x + y = 2a$ $(a > 0)$.
   Ans. $\int_0^a[\int_0^{2a-2x} dy]\, dx$

   (d) The y-axis, the line $y = 3x$, and the line $y = 6$.
   Ans. $\int_0^2[\int_{3x}^{6} dy]\, dx$

   (e) The x-axis, the curve $y = e^{-x}$, and the lines $x = 0$, $x = a$ $(a > 0)$.
   Ans. $\int_0^a[\int_0^{e^{-x}} dy]\, dx$

   (f) The parabolas $y = x^2$ and $y = 2x - x^2$.
   Ans. $\int_0^1[\int_{x^2}^{2x-x^2} dy]\, dx$

2. (a) Evaluate $\int\int_D (x - y^2) dxdy$, where D is the region $\{(x, y)|0 \le x \le 4,\ \sqrt{x} \le y \le 2\ \}$.
   Ans. $\int_0^4[\int_{\sqrt{x}}^{2}(x - y^2)\, dy]\, dx$

   (b) Evaluate $\int\int_D (1 - \sin \pi x) y dxdy$, where D is the region $\{(x, y)|0 \le x \le 1,\ 0 \le y \le x\}$.
   Ans. $\int_0^1[\int_0^x (1 - \sin \pi x) y\, dy]\, dx$

   (c) Evaluate $\int\int_D (x^2 + y) dxdy$, where D is the region $\{(x, y)|-3 \le y \le 2,\ 0 \le x \le y^2\}$.
   Ans. $\int_{-3}^{2}[\int_0^{y^2}(x^2 + y)\, dx]\, dy$

   (d) Evaluate $\int\int_D (x^2+y^2) dxdy$, where D is the region bounded by the positive x-axis, the positive y-axis, and the line $3x + y = 9$.
   Ans. $\int_0^3[\int_0^{9-3x}(x^2 + y^2)\, dy]\, dx$

   (e) Evaluate $\int\int_D y^2\sqrt{x} dA$, where D is the region $\{(x, y)|x > 0,\ x^2 < y < 10 - x^2\}$.
   Ans. $\int_0^{\sqrt{5}}[\int_{x^2}^{10-x^2} y^2\sqrt{x}\, dy]\, dx$

3. Evaluate $\int\int\int_B dxdydz$, where B is the region bounded by the coordinate planes and the plane $x + y + z = 1$.
   Ans. $\int_0^1[\int_0^{1-x}(\int_0^{1-x-y} dz)dy]dx$

4. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$.
   Ans. $\int_{-a}^{a}[\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}(\int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} dz)dy]dx$

5. Evaluate

   (a) $\int_0^1\int_0^{x^2}\int_0^{xy^3} 18x^3y^2z dzdydx$
   Ans. $\frac{1}{24}$

   (b) $\int_0^1\int_{y^2}^{1}\int_0^{1-x} x dzdxdy$
   Ans. $\frac{4}{35}$

   (c) $\int_0^2\int_0^{\pi}\int_0^{\ln 4} x^3 \cos\frac{y}{2} e^z dzdydx$
   Ans. 24

(d) $\int_0^1 \int_0^{\sqrt{3}z} \int_0^{\sqrt{3(y^2+z^2)}} xyz\sqrt{x^2+y^2+z^2}dxdydz$
Ans. $\frac{31}{15}$

(e) $\int_0^{\sqrt{\frac{\pi}{2}}} \int_x^{\sqrt{\frac{\pi}{2}}} \int_1^3 \sin y^2 dzdydx$
Ans. $2\int_0^{\sqrt{\frac{\pi}{2}}} \int_0^y \sin y^2 dxdy = 1$

6. Evaluate $\int\int\int_W zdxdydz$, where W is the region bounded by the four planes $x=0$, $y=0$, $z=0$, $z=1$, and the cylinder $x^2+y^2=1$, with $x \geq 0$, $y \geq 0$.
Ans. $\int_0^1 [\int_0^{\sqrt{1-x^2}} (\int_0^1 zdz)dy]dx$

7. Evaluate $\int\int\int_W ze^{x+y}dxdydz$, where $W=[0,1]\times[0,1]\times[0,1]$.
Ans. $\frac{(e-1)^2}{2}$

8. Evaluate $\int\int\int_W (x^2+y^2+z^2)dxdydz$, where W is the region bounded by $x+y+z=a$ $(a>0)$, $x=0$, $y=0$, and $z=0$.
Ans. $\int_0^a [\int_0^{a-x} (\int_0^{a-x-y} (x^2+y^2+z^2)dz)dy]dx = \frac{a^5}{20}$

9. Set up a triple integral for the volume of each of the following solid regions.

(a) The region in the first octant bounded above by the cylinder $z=1-y^2$ and lying between the vertical planes given by $x+y=1$ and $x+y=3$
Ans. $\int_0^1 [\int_{1-y}^{3-y} (\int_0^{1-y^2} dz)dx]dy$

(b) The upper hemisphere given by $z=\sqrt{1-x^2-y^2}$.
Ans. $\int_{-1}^1 [\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (\int_0^{\sqrt{1-x^2-y^2}} dz)dy]dx$

(c) The region bounded below by the paraboloid $z=x^2+y^2$ and above by the sphere $x^2+y^2+z^2=6$.
Ans. $\int_{-\sqrt{2}}^{\sqrt{2}} [\int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} (\int_{x^2+y^2}^{\sqrt{6-x^2-y^2}} dz)dy]dx$

10. Use the triple integration to find the volumes of the given regions.

(a) The region in the first octant bounded by the cylinder $x=4-y^2$ and the planes $y=z$, $x=0$, $z=0$.
Ans. $\int_0^2 [\int_0^{4-y^2} (\int_0^y dz)dx]dy$

(b) The region above the xy-plane bounded by the surfaces $z^2=16y$, $z^2=y$, $y=x$, $y=4$, and $x=0$.
Ans. $\int_0^4 [\int_0^y (\int_{\sqrt{y}}^{4\sqrt{y}} dz)dx]dy$

(c) The region bounded by the paraboloids $z=8-x^2-y^2$ and $z=x^2+3y^2$.
Ans. $\int_{-\sqrt{2}}^{\sqrt{2}} [\int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} (\int_{x^2+y^2}^{8-x^2-y^2} dz)dx]dy$

(d) The region bounded by the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$
Ans. $\int_{-b}^b [\int_{-a\sqrt{1-\frac{y^2}{b^2}}}^{a\sqrt{1-\frac{y^2}{b^2}}} (\int_{-c\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}}^{c\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}} dz)dx]dy$

(e) The region bounded by the cylinder $z=4-y^2$ and the paraboloid $z=x^2+3y^2$.
Ans. $\int_{-1}^1 [\int_{-2\sqrt{1-y^2}}^{2\sqrt{1-y^2}} (\int_{x^2+3y^2}^{4-y^2} dz)dy]dx$

(f) The region bounded by the coordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
Ans. $\int_0^b [\int_0^{a(1-\frac{y}{b})} (\int_0^{1-\frac{x}{a}-\frac{y}{b}} dz)dx]dy$

(g) The region bounded by the cylinder $x^2+y^2=4x$, the xy-plane and the paraboloid $4z=x^2+y^2$.
Ans. $\int_0^4 [\int_{-\sqrt{4x-x^2}}^{\sqrt{4x-x^2}} (\int_0^{\frac{1}{4}(x^2+y^2)} dz)dy]dx$