QUIZ #2

Math 252 October 8, 2009

**KEY**

**Show your work for full credit.**

1. (5 points) Change the order of integration of
$\int_0^1 [\int_0^{2x} \; y\, e^{x^3}\, dy\,]\, dx.$

   The given integral extends over the region R:
   R : $0 \le x \le 1$, $0 \le y \le 2x$.
   We sketch a graph of R (it is a right triangle with vertices at $(0, 0), (1, 0)$, and $(1, 2)$ ) to see that R can also be given by
   R: $0 \le y \le 2$, $\frac{y}{2} \le x \le 1$.
   Thus, the integral in the reverse order of integration is given by
   $\int_0^2 [\int_{\frac{y}{2}}^{1} \; y\, e^{x^3}\, dx\,]\, dy.$

2. (5 points) Set up a double integral that computes the area of the region bounded by the curves $y = x^2$ and $y = 2 - x$.

   The points of intersection are obtained by equating the y values of the two curves; that is, $x^2 = 2 - x$. Thus, $x^2 + x - 2 = 0 \implies (x + 2)(x - 1) = 0$. Hence $x = 1$ or $x = -2$. Therefore, $(1, 1)$, and $(-2, 4)$ are the points of intersection. Use these points to sketch a graph and note that the region R bounded by the two curves is given by
   (1) $-2 \le x \le 1$, $x^2 \le y \le 2 - x$
   or
   (2) $0 \le y \le 1$, $-\sqrt{y} \le x \le \sqrt{y}$ or $1 \le y \le 4$, $-\sqrt{y} \le x \le 2 - y$.
   Hence, the two possible answers are
   (1) $\int_{-2}^{1} [\int_{x^2}^{2-x} \; dy\,]\, dx.$
   and
   (2) $\int_0^1 [\int_{-\sqrt{y}}^{\sqrt{y}} \; dx\,]\, dy + \int_1^4 [\int_{-\sqrt{y}}^{2-y} \; dx\,]\, dy$