

Answer the following. Show your work for full credit.

1. Compute the following limits:

(a) (3 points) $\lim_{x \rightarrow 5^-} \frac{|x-5|}{x-5}$

Here $x < 5$. For $x < 5$, $|x - 5| = -(x - 5)$.

Thus, $|x - 5| = -(x - 5)$. So, $\lim_{x \rightarrow 5^-} \frac{|x-5|}{x-5} = \lim_{x \rightarrow 5^-} \frac{-(x-5)}{x-5} = \lim_{x \rightarrow 5^-} (-1) = -1$.

(b) (3 points) $\lim_{x \rightarrow 3^-} \frac{2}{x-3}$

Since $\frac{2}{3-3} = \frac{2}{0}$, this is an infinite limit. We have to determine if the limit is ∞ or $-\infty$. Here,

$x < 3$. Choose $x = 2.9$. Since $\frac{2}{2.9-3} = -20$ is a negative number, $\lim_{x \rightarrow 3^-} \frac{2}{x-3} = -\infty$.

2. Let

$$f(x) = \begin{cases} 5x + 1 & \text{if } x < 3 \\ 3x - 2 & \text{if } x \geq 3 \end{cases}$$

Find

(a) (2 points) $\lim_{x \rightarrow 3^-} f(x)$

Here, $x < 3$. In this case, $f(x) = 5x + 1$. Thus, $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (5x + 1) = 5(3) + 1 = 15 + 1 = 16$.

(b) (2 points) $\lim_{x \rightarrow 3^+} f(x)$

Here, $x > 3$. In this case, $f(x) = 3x - 2$. Thus, $\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (3x - 2) = 3(3) - 2 = 9 - 2 = 7$.