

Show your work for full credit.

1. Find the following limits:

(a) (4 points)  $\lim_{x \rightarrow -1} \frac{x+1}{x^2+x}$

Note that the substitution  $x = -1$  gives  $\frac{0}{0}$ .

$$\begin{aligned} & \lim_{x \rightarrow -1} \frac{x+1}{x^2+x} \\ &= \lim_{x \rightarrow -1} \frac{x+1}{x(x+1)} \\ &= \lim_{x \rightarrow -1} \frac{1}{x} \quad (\text{since } x \neq -1) \\ &= \frac{1}{-1} \\ &= -1. \end{aligned}$$

(b) (3 points)  $\lim_{x \rightarrow 0} \frac{\frac{1}{x+1}-1}{x}$

Note that the substitution  $x = 0$  gives  $\frac{0}{0}$ .

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\frac{1}{x+1}-1}{x} = \\ &= \lim_{x \rightarrow 0} \frac{\frac{1}{x+1}-1 \cdot \frac{x+1}{x+1}}{\frac{x}{x+1}} = \\ &= \lim_{x \rightarrow 0} \frac{\frac{1-(x+1)}{x+1}}{\frac{x}{x+1}} = \\ &= \lim_{x \rightarrow 0} \frac{-x}{x} = \\ &= \lim_{x \rightarrow 0} \frac{-1}{x+1} = \\ &= \frac{-1}{0+1} = \\ &= \frac{-1}{1} \\ &= -1. \end{aligned}$$

(c) (3 point)  $\lim_{x \rightarrow 0} \frac{\sin 5x}{3x}$

Note that the substitution  $x = 0$  gives  $\frac{0}{0}$ .

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sin 5x}{3x} = \\ & \lim_{x \rightarrow 0} \frac{1}{3} \frac{\sin 5x}{x} = \\ & \lim_{x \rightarrow 0} \frac{1}{3} \frac{\sin 5x}{x} = \\ & \lim_{x \rightarrow 0} \frac{1}{3} \frac{\sin 5x}{x} \cdot \frac{5}{5} = \\ & \lim_{x \rightarrow 0} \frac{5}{3} \frac{\sin 5x}{5x} = \\ & \frac{5}{3} \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} = \\ & \frac{5}{3} (1), \text{ since } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \\ &= \frac{5}{3}. \end{aligned}$$